

Kapovitch's class, Wed Jan 7: More basic Morse theory

January-07-15 11:08 AM

Review of Morse theory.

$F: M^n \rightarrow \mathbb{R}$ is a "Morse Function" if all its critical points are non-degenerate, and non are on the boundary.

$\text{index}(p) = \text{index of Hess}(F)|_p = \# \text{ of negative eivals.}$

Morse Lemma: Locally, if $\text{ind} = \lambda$,

$$F(x) = -x_1^2 - \dots - x_\lambda^2 + x_{\lambda+1}^2 + \dots + x_n^2$$

PF sketch: Write $F(x) = \sum_{i,j} x_i x_j h_{ij}(x)$

where $h_{ij}(x)$ is symmetric.

claim For any $r \geq 0$ in some local coords

$$F(x) = \pm x_1^2 \pm \dots \pm x_r^2 + \sum_{i,j > r} \tilde{h}_{ij}(x) x_i x_j$$

by induction, if

$$F(x) = \pm x_1^2 \pm \dots \pm x_{r-1}^2 + \sum_{i,j > r-1} h_{ij}(x) x_i x_j$$

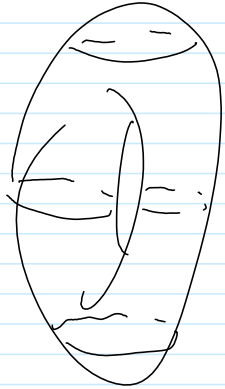
by a lin. change of coords $h_{rr}(0) \neq 0$;

now put $y_i = \begin{cases} x_i & i \neq r \\ \sqrt{|h_{rr}(x)|} \cdot \left(x_r + \sum_{j > r} x_j \frac{h_{jr}(x)}{h_{rr}(x)} \right) & i = r. \end{cases}$

claim On any manifold there are plenty of

Morse functions.

• • • • Then the usual discussion of Morse Theory



etc. . . .