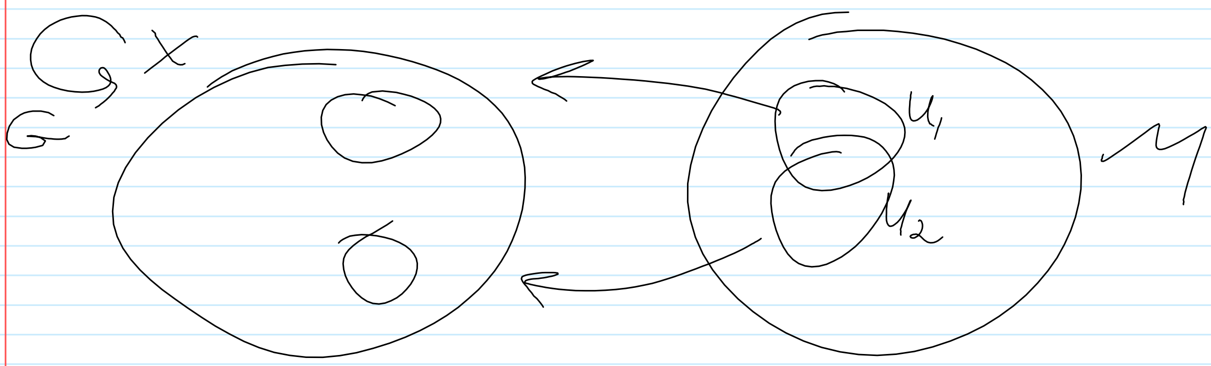
Kapovitch's class, Mon Jan 19: Rafi on "Geometric Structures"

January-19-15 11:04 AM

By Kasra Rafi

Thm A compact closed 3-manifold can be decomposed? into pieces each of which has a geometric structure?

An (X,G)-structure



s.t. transition functions are in G.

precisely, M is covered with $\varphi_\alpha : U_\alpha \to X$ s.t. $\varphi_\beta \varphi_\alpha^{-1}$ is in G.

Examples $(\mathbb{R}^n, \text{homeo})$, $(\mathbb{C}, \text{holo}, \ldots)$

X – simply-connected Riemannian mfld, G transitive: an (X,G) structure is called "a geometric structure"

In 2D $X = S^2, \mathbb{R}^2, \mathbb{D}^2 = \mathbb{H}^2$

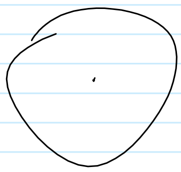
"spherical geom" "hyperbolic geom"

"spherical geom" $K=1$

"Euc geom" $K=0$

"hyperbolic geom" $K=-1$ (area of triangles $= \pi-\alpha-\beta-\gamma$)

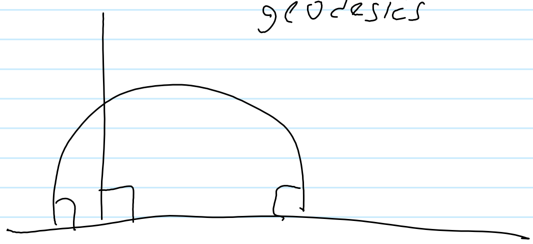
$\mathbb{H}^2$: upper half plane w/ $\frac{dx^2+dy^2}{y^2}$



$\frac{dx^2+dy^2}{2(1-r^2)^2}$

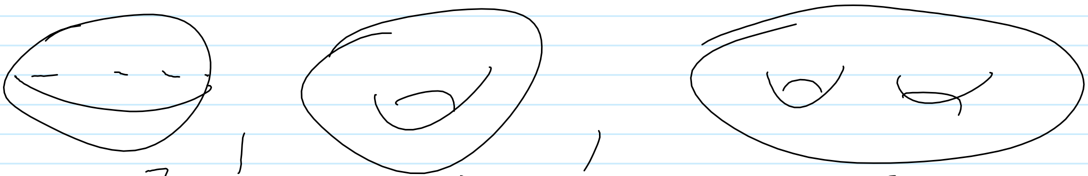
Poincare disk

geodesics

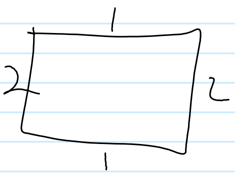


<u>Thm</u> Every 2D manifold has a geometric structure.

<u>Pf</u> Topologically,

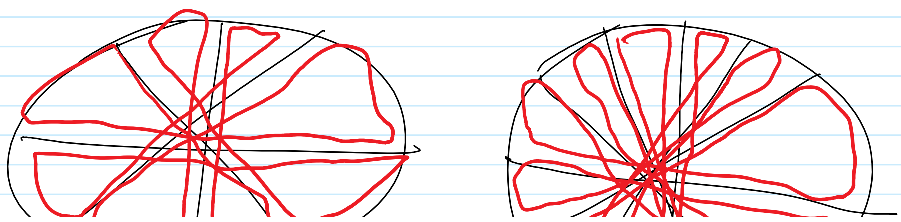


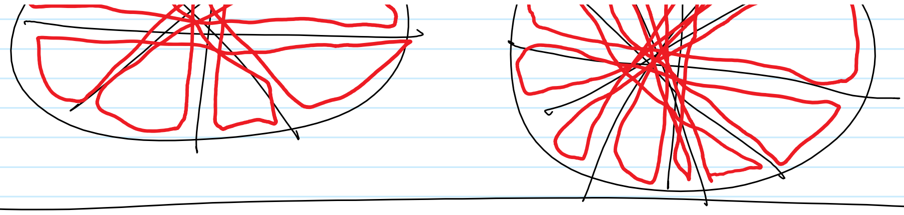
S^2, T^2, Σ_g



Σ_g = a gluing of a $4g$-gon along opposite edges ?

Aside:





In 3D, the geometries are

$\mathbb{E}^3$, $\mathbb{H}^3$, $\mathbb{S}^3$, $\mathbb{S}^2\times\mathbb{R}$, $\mathbb{H}^2\times\mathbb{R}$,

Nil-geom, Sol-geom, $\widetilde{SL_2\mathbb{R}}$-geom.