Huizenga@Colloq: Interpolation problems in algebraic geometry

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Lagrangian Interpolation

$p_1 \dots p_n \in \mathbb{C}$ "points" (distinct)

$q_1 \dots q_n \in \mathbb{C}$ "values"

$\Rightarrow \exists!$ f poly of deg $\le n-1$ s.t. $f(p_i)=q_i$

Proof $ev : S_{n-1} \longrightarrow \mathbb{C}^n$

$f \longmapsto (f(p_1) \dots f(p_n))$

is 1-1 hence onto

Multivariable interpolation:

$p_1 \dots p_n \in \mathbb{C}^r$

What is the rank of

$$ev : S_m \longrightarrow \mathbb{C}^n$$

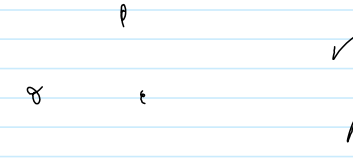
$\uparrow$

poly's deg $\le m$

Equivalently, how many indep. conditions does vanishing at n impose on $S_m \subset_6$

Example $n=3$ pts in plane

$r=2$ $m=1$



Def'n a collection of pts "has interpolation

in deg m if ev_m has max rank.

Prop General pts $p_1 \dots p_n$ have interpolation in deg m.

pf Pick p_1 arbitrarily.

p_2 chosen s.t. not all f that vanish at p_1 also vanish at p_2.

keep going ....

until every poly that vanishes on $p_1 \dots p_k$ is the zero poly. Past this no matter what choice.

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(1) What happens at special configs?

(2) What if we prescribe derivatives as well? 20 minutes.

(3) Impose general zeros on sections of vector bundles.

Motivation Birational geometry of Hilbert schemes of points in $\mathbb{P}^2$