Tangles, Wheels, Balloons

Abstract. I will describe an invariant of tangles with values in wheels, almost generalize it to some balloons, and then tell you why I care. Spoilers: tangles are you know what, wheels are linear combinations of cyclic words in some alphabet, balloons are 2-knots, and one reason I care is because quantum field theory predicts more than I can actually get (but also less).

ingreen: computable, non-commutative

Why I like non-commutative?

$\dim \mathbb{Q}[x,y]_d \sim d \ll 2^d = \dim FA[x,y]_k$

Why I like computable?

* π_1 isn't computable.
* It's my personal perversion

preliminaries

$FL(x_1 \ldots x_n) = \text{trees}/IHX$

$FA(FL(x_1 \ldots x_n)) \to FL(x_1 \ldots x_n)$

$CW(x_1 \ldots x_n)$

$CW(FL(x_1 \ldots x_n)) \to CW(x_1 \ldots x_n)$

Claim 2-in-1-out connected directed trivalent graphs /AS, IHX
$= CW(1 \ldots n)$

with ends in $\{1 \ldots n\}$

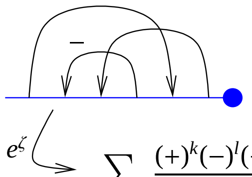
A word about Gauss diagrams

The def of the invariant as in next page

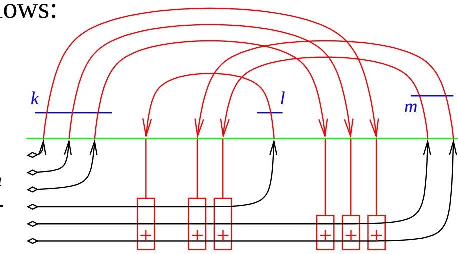
proof of invariance

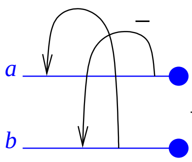
All the connections...

Theorem 1 (with Cattaneo (credit, no blame)). In the ribbon case, e^ζ can be computed as follows:

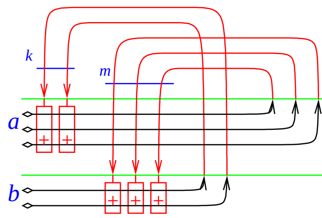


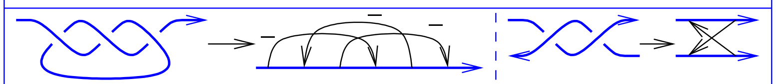
$$e^\zeta \to \sum_{k,l,m\geq 0} \frac{(+)^k(-)^l(+)^m}{k!l!m!}$$

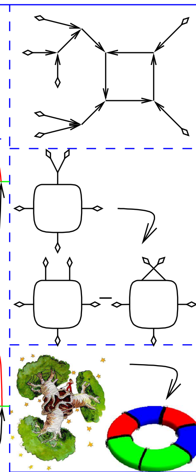




$$\xrightarrow{e^\zeta} \sum_{k,m\geq 0} \frac{(+)^k(-)^m}{k!m!}$$







Theorem 2. Using Gauss diagrams to represent knots and T-component pure tangles, the above formulas define an invariant in $CW(FL(T)) \to CW(T)$, "cyclic words in T".

• Agrees with BN-Dancso [**?**] and with [BN2]. • In-practice computable! • Vanishes on braids. • Extends to w. • Contains Alexander. • The "missing factor" in Levine's factorization [Le] (the rest of [Le] also fits, hence contains the MVA). • Related to / extends Farber's [Fa]? • Should be summed and categorified.

References.

[BN2] D. Bar-Natan, *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant*, `http://www.math.toronto.edu/~drorbn/papers/KBH/`, arXiv:1308.1721.

[CR] A. S. Cattaneo and C. A. Rossi, *Wilson Surfaces and Higher Dimensional Knot Invariants*, Commun. in Math. Phys. **256-3** (2005) 513–537, arXiv:math-ph/0210037.

[Fa] M. Farber, *Noncommutative Rational Functions and Boundary Links*, Math. Ann. **293** (1992) 543–568.

[Le] J. Levine, *A Factorization of the Conway Polynomial*, Comment. Math. Helv. **74** (1999) 27–53, arXiv:q-alg/9711007.

[Wa] T. Watanabe, *Configuration Space Integrals for Long n-Knots, the Alexander Polynomial and Knot Space Cohomology*, Alg. and Geom. Top. **7** (2007) 47-92, arXiv:math/0609742.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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