

Chord Diagrams and an - ✓

Video, handout, links and more at <http://www.math.toronto.edu/~drorbn/Talks/Fields-1411/> Dror Bar-Natan: Talks: Fields-1411: **Finite Type Invariants of Doodles, 1**

Abstract. I will describe my former student's Jonathan Zung work on finite type invariants of "doodles", plane curves modulo the second Reidemeister move but not modulo the third. We use a definition of "finite type" different from Arnold's and more along the lines of Goussarov's "Interdependent Modifications", and come to a conjectural combinatorial description of the set of all such invariants. We then describe how to construct many such invariants (though perhaps not all) using a certain class of 2-dimensional "configuration space integrals". *An unfinished project!*

Doodles. $\mathcal{K} = \mathcal{K}_0 = \mathbb{Q} \langle \text{doodles} \rangle / \langle \text{R2} \rangle$ Easy to classify! yet not R1/R3

Prior Art. Arnold [Ar] first studied doodles within his study of plane curves and the "strangeness" St invariant. Vassiliev [Va1, Va2] defined finite type invariants in a different way, and Merkov [Me] proved that they separate doodles.

Goussarov Finite-Type. $\mathcal{K}_n = \langle \text{doodles} \rangle / \langle \text{ring}, \text{join} \rangle$ doodles and detours (dnd's)

Def. V is of type n if it vanishes on \mathcal{K}_{n+1} . $(\mathcal{K}_0/\mathcal{K}_{n+1})^* \leftrightarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

Knots in 3D. **2-Knots in 4D.** The reason I care!

Goals. • Describe $\mathcal{A}_n := \mathcal{K}_n/\mathcal{K}_{n+1}$ using diagrams/relations. • Get many or all finite type invariants of doodles using configurations space integrals. • Do these come from a TQFT? • See if \mathcal{A}_n has a "Lie theoretic" (tensors/relations) meaning. • See if/how Arnold's St and the Merkov invariants integrate in.

Important Example.

Summary Diagram. MC: (Multi-Commutator) relations.

$$\begin{array}{ccccccc} & & \mathcal{K} & & & & \\ & & \Downarrow Z & & & & \\ \mathcal{D}^c / \text{MC} := \mathcal{A}^c & \xrightarrow{\pi} & \text{gr} \mathcal{K} & \xrightarrow{\text{gr} Z} & \mathcal{A}' := \mathcal{D}' / \text{FDR} & \xrightarrow{?} & \mathcal{A}^c \\ & \searrow \tau & & & & & \end{array}$$

...ation space integrals

Configuration space integrals,
 Feynman Diagrams
 and a

Video, handout, links and more at
<http://www.math.toronto.edu/~drorbn/Talks/Fields-1411/>
 A Lower Bound on $(\mathcal{K}_0/\mathcal{K}_{n+1})^*$.

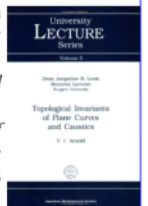
Dror Bar-Natan: Talks: Fields-1411: **Finite Type Invariants of Doodles, 2**

An unfinished project!

copy Zung w/ some
 colour coding.

References. The root, of course, is [Ar]. Further references on doodles include [Kh, FT, Me, Ta, Va1, Va2]. On Goussarov finite-type: [Go, BN].

- [Ar] V.I. Arnold, *Topological Invariants of Plane Curves and Caustics*, American Mathematical Society, 1994.
- [BN] D. Bar-Natan, *Bracelets and the Goussarov filtration of the space of knots, Invariants of knots and 3-manifolds (Kyoto 2001)*, Geometry and Topology Monographs **4** 1–12, [arXiv:math.GT/0111267](https://arxiv.org/abs/math.GT/0111267).
- [FT] R. Fenn and P. Taylor, *Introducing Doodles*, in *Topology of Low-Dimensional Manifolds, Proceedings of the Second Sussex Conference, 1977*, Springer 1979.
- [Go] M. Goussarov, *Interdependent modifications of links and invariants of finite degree*, *Topology* **37-3** (1998) 595–602.
- [Kh] M. Khovanov, *Doodle Groups*, *Trans. Amer. Math. Soc.* **349-6** (1997) 2297–2315.
- [Me] A.B. Merkov, *Vassiliev Invariants Classify Plane Curves and Doodles*, *Sbornik: Mathematics* **194-9** (2003) 1301.
- [Ta] S. Tabachnikov, *Invariants of Smooth Triple Point Free Plane Curves*, *Jour. of Knot Theory and its Ramifications* **5-4** (1996) 531–552.
- [Va1] V.A. Vassiliev, *On Finite Order Invariants of Triple Point Free Plane Curves*, 1999 preprint, [arXiv:1407.7227](https://arxiv.org/abs/1407.7227).
- [Va2] V.A. Vassiliev, *Invariants of Ornaments*, *Adv. in Soviet Math.* **21** (1994) 225–262.



“God created the knots, all else in topology is the work of mortals.”

Leopold Kronecker (modified)

www.katlas.org

