A funny Lie bi-algebra

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Let $\mathfrak{g}$ be a Lie algebra, D a fixed element in it, and set $\mathfrak{g}_1 = \mathfrak{g} \oplus \langle C \rangle$, where C is a new central element.

Define $\delta: \mathfrak{g}_1 \to \mathfrak{g}_1 \otimes \mathfrak{g}_1$ by

$$\delta C = 0$$

$X \in \mathfrak{g}$ $\quad \delta X = [D, X] \otimes C - C \otimes [D, X].$

<u>Claim.</u> $(\mathfrak{g}_1, [,], \delta)$ is a Lie bi-algebra.

Anti-symmetry is obvious.

Co-Jacobi:

$$(\delta \otimes 1)\delta(X) = (\delta \otimes 1)([D,X] \otimes C - C \otimes [D,X])$$

$$= [D,[D,X]] \otimes C \otimes C - C \otimes [D,[D,X]] \otimes C$$

this clearly vanishes under cyclic summation.

Co-cycle condition:

$$\delta[X,Y] \stackrel{?}{=} [\delta X, Y \otimes 1 + 1 \otimes Y] + [X \otimes 1 + 1 \otimes X, \delta Y]$$

check:

$$\delta[X,Y] = [D,[X,Y]] \otimes C - C \otimes [D,[X,Y]]$$

$$[\delta X, Y \otimes 1 + 1 \otimes Y] = [[D,X] \otimes C - C \otimes [D,X], Y \otimes 1 + 1 \otimes Y]$$

$$= [[D,X],Y] \otimes C - C \otimes [[D,X],Y]$$

$$[X \otimes 1 + 1 \otimes X, \delta Y] = [X \otimes 1 + 1 \otimes X, [D,Y] \otimes C - C \otimes [D,Y]]$$

$$= [X,[D,Y]] \otimes C - C \otimes [X,[D,Y]]$$

checks!

Added Nov 9: This may be a special case of a general "twisting" procedure that given a Lie bialgebra and an element inside it constructs a new Lie bialgebra.

Can this be iterated?

Added Nov 13: Meinrenken: this is probably related to "Medina-Revoy double extensions".