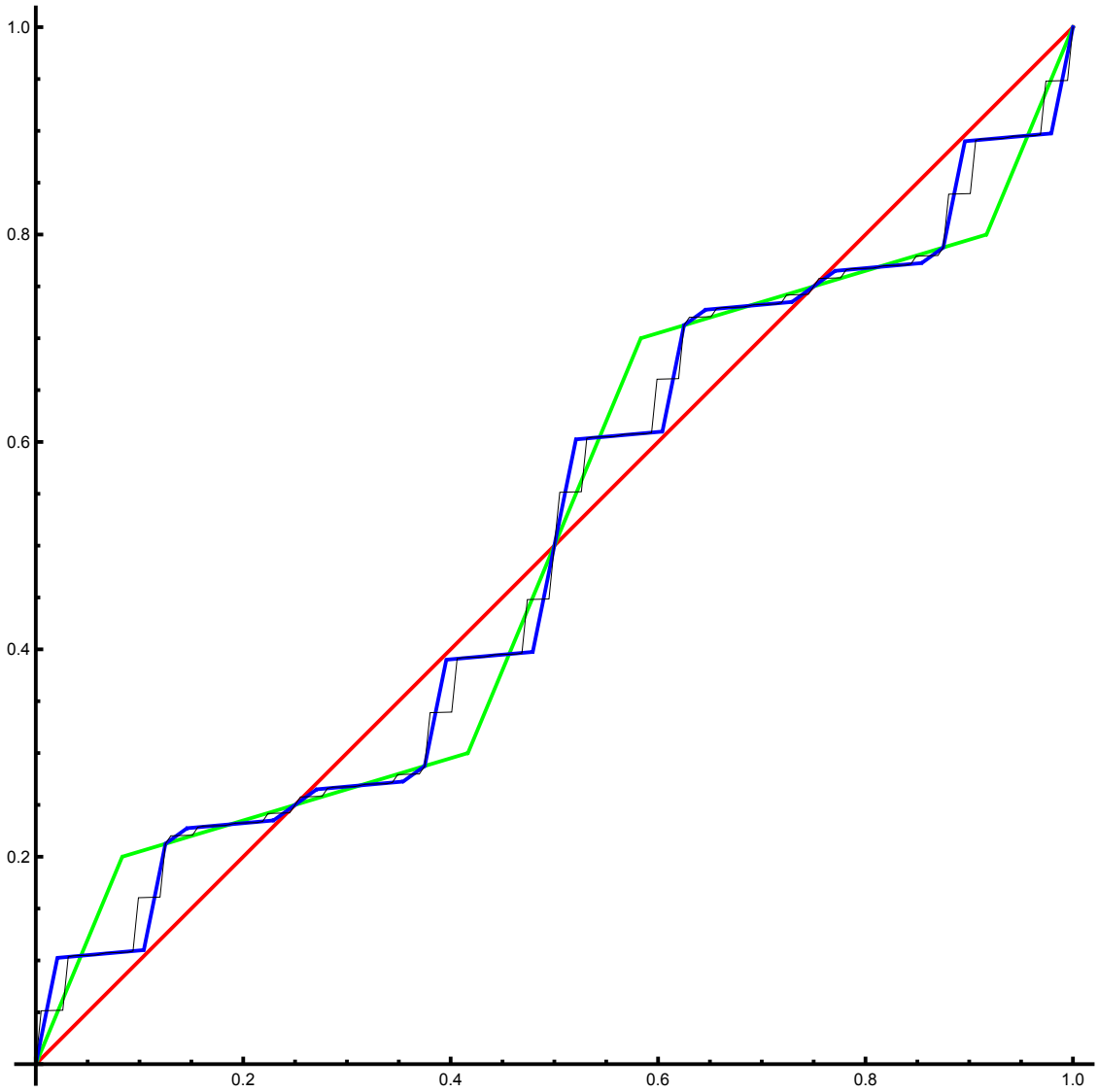
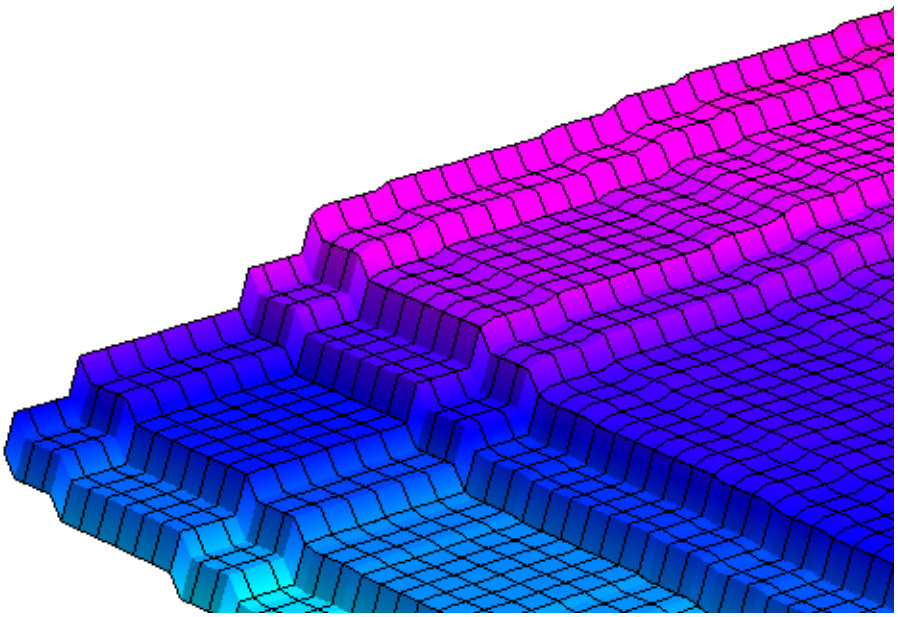
Hilbert's 13th Problem

Pensieve Header: Hilbert's 13th problem - Step 2 to level 3.

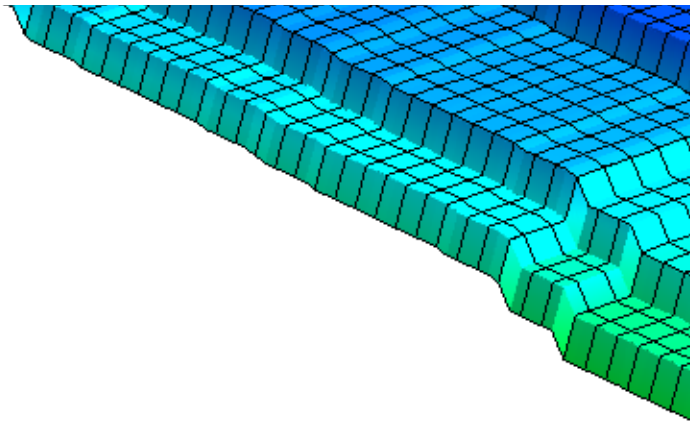
```
<< "../../2009-11/Hilbert13th-Program.m"

ϕ1 := Phi[Identity, 2, 0.3, 2 / 3];
ϕ2 := Phi[ϕ1, 8, 0.3^2, 2 / 3];
ϕ3 := Phi[ϕ2, 32, 0.3^3, 2 / 3];
Step2phis = Plot[{x, ϕ1[x], ϕ2[x], ϕ3[x]}, {x, 0, 1},
  PlotPoints → 1279, ColorFunction → Automatic,
  AxesStyle → Thick, PlotStyle → {Directive[Red, Thick],
    Directive[Green, Thick], Directive[Blue, Thick], Directive[Black]}
 ]
```



```
Timing[
 L3Step2Cascade = Rasterize[
   Plot3D[ϕ3[x] + λ * ϕ3[y], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 1279, Mesh → 95, ViewPoint → {-2, -2, 1},
    NormalsFunction → None, Boxed → False, Axes → None, ImageSize → 2400
   ], ImageSize → 2400, RasterSize → 2400
  ]
]
```



{2586.709761,



```
Export[
 "L3Step2Cascade.png",
 ImageCrop[L3Step2Cascade]
]
```

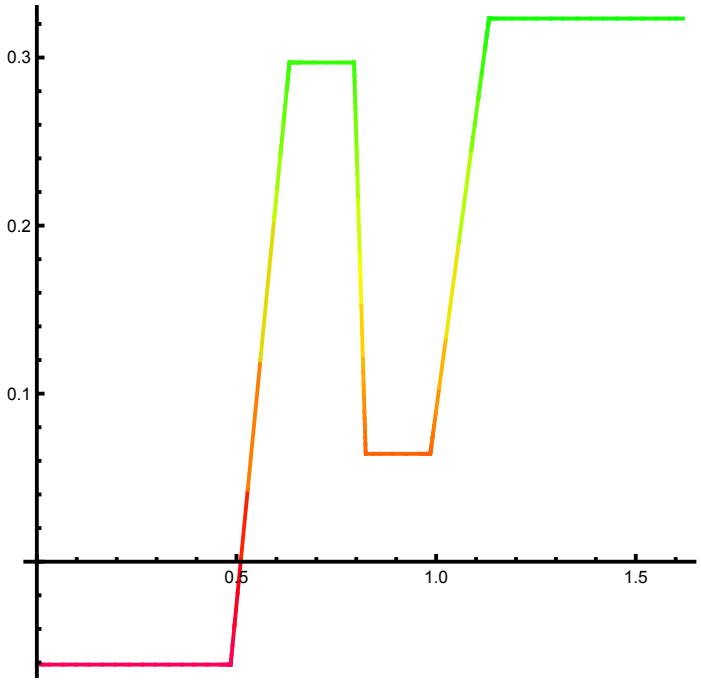
L3Step2Cascade.png

```
phi1 := Phi[Identity, 2, 0.3, 2 / 3];
phi2 := Phi[phi1, 12, 0, 0.8];
phi3 := Phi[ϕ, ϕ0 → phi1, Subdivisions → 12, Slope → 0, FillFactor → 0.8];
g1 = G[f, phi1];
g2 = G[f, phi2];
```

```
Step2G1 = Plot[
  g1[z], {z, 0, 1 + λ},
  AxesStyle → Thick, PlotStyle → Thick
 ]
```

InterpolatingFunction::dmval :

Input value {0.0000330541} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫



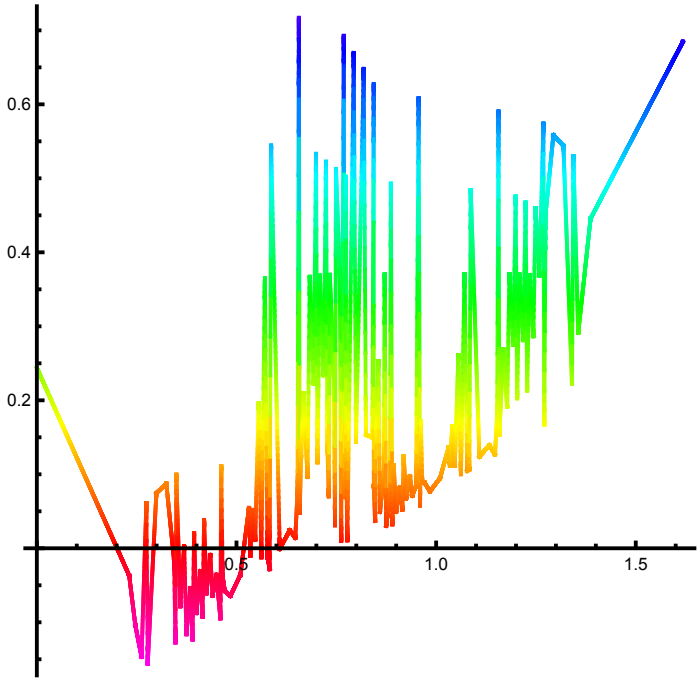

```
Export[
 "Step2G1.png",
 ImageCrop[Step2G1]
]
```

```
Step2G1.png
```

```
Step2G2 = Plot[
  g2[z], {z, 0, 1 + λ},
  PlotPoints → 5000, AxesStyle → Thick, PlotStyle → Thick
 ]
```

InterpolatingFunction::dmval :

Input value $\{3.23995 \times 10^{-7}\}$ lies outside the range of data in the interpolating function. Extrapolation will be used. >>




```
Export[
 "Step2G2.png",
 ImageCrop[Step2G2]
]
```

Step2G2.png

```
Timing[
 Step2CascadeWithG1 = Rasterize[
   Plot3D[phi2[x] + λ * phi2[y], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 301, Mesh → 23, ViewPoint → {-2, -2, 1}, NormalsFunction → None,
    ColorFunction → (Hue[g1[#3]] &), Boxed → False, Axes → None
   ]
  ]
]
```

InterpolatingFunction::dmval :

Input value $\{6.47214 \times 10^{-8}\}$ lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

InterpolatingFunction::dmval :

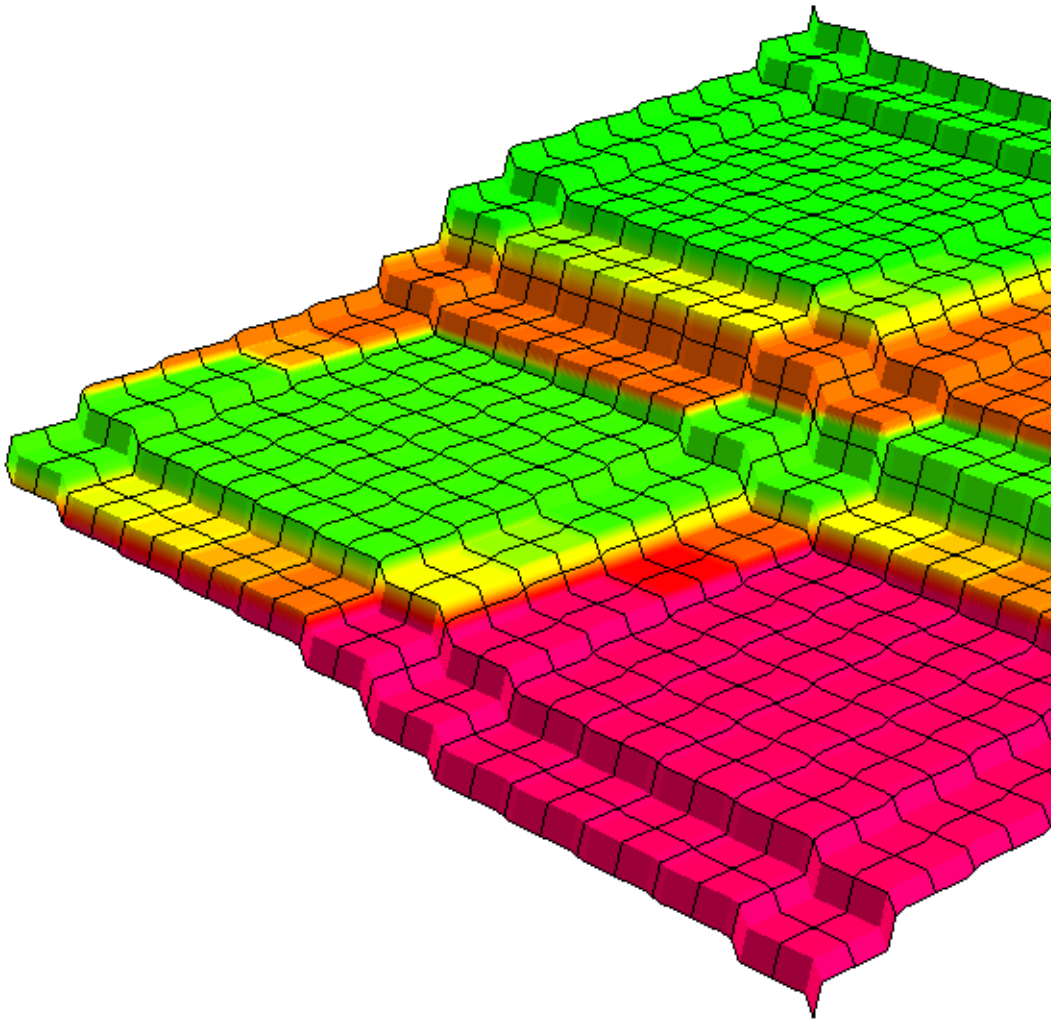
Input value {0.0400001} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

InterpolatingFunction::dmval :

Input value {0.0800001} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫

General::stop : Further output of InterpolatingFunction::dmval will be suppressed during this calculation. ≫

{135.824,

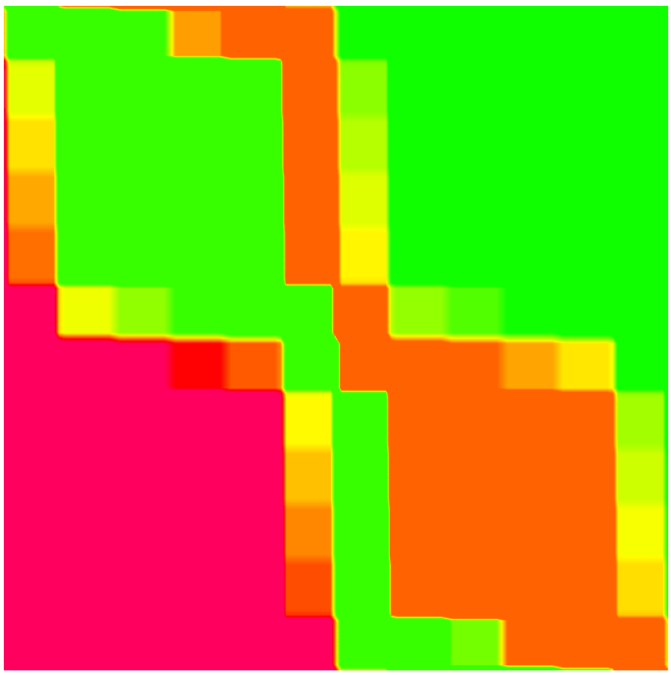


```
Export[
 "Step2CascadeWithG1.png",
 ImageCrop[Step2CascadeWithG1]
]
```

Step2CascadeWithG1.png

```
Timing[
 Step2DensityWithG1 = Rasterize[
   DensityPlot[
    g1[phi2[x] + λ * phi2[y]], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 301, Frame → False
   ]
  ]
]
```

InterpolatingFunction::dmval :
Input value {0.0000647214} lies outside the range of data in the interpolating function. Extrapolation will be used. >>

{84.4052,  }

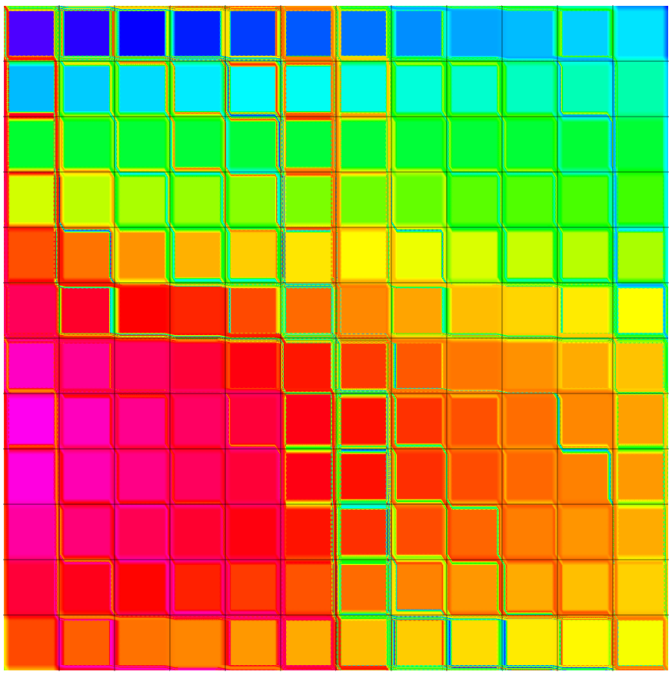
```
Export[
 "Step2DensityWithG1.png",
 ImageCrop[Step2DensityWithG1]
]
```

Step2DensityWithG1.png

```
Timing[
 Step2DensityWithG2 = Rasterize[
   DensityPlot[
    g2[phi2[x] + λ * phi2[y]], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 359, Mesh → 11, Frame → False
   ]
  ]
]
```

InterpolatingFunction::dmval :

Input value {0.0000542358} lies outside the range of data in the interpolating function. Extrapolation will be used. >>

{275.746,  }

```
Export[
 "Step2DensityWithG2.png",
 Step2DensityWithG2
]
```

Step2DensityWithG2.png