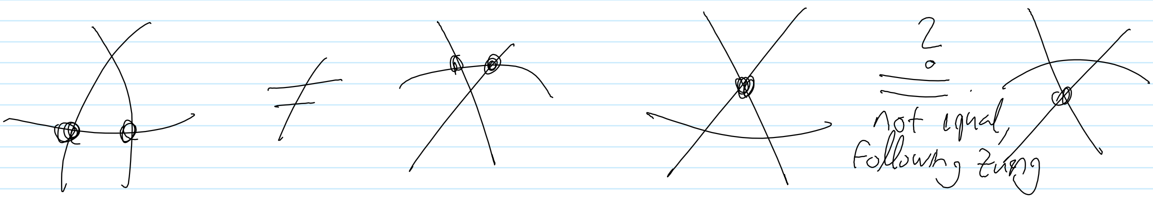
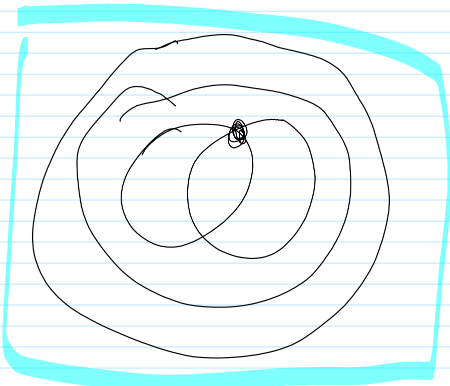
Virtual Doodles

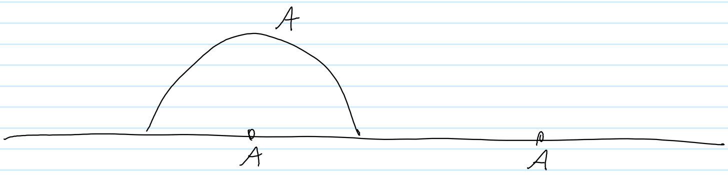
October-21-14 8:27 PM

Jonathan's Comment. It seems that the configuration space integrals we defined are more naturally invariants of virtual doodles. Virtual doodles are doodles with some ordinary crossings and some virtual crossings, with the relation that triple points having three virtual crossings are allowed. (Caution: virtual doodles are not Gauss diagrams modulo Reidemeister 2.)
The integrals we defined are invariants for virtual doodles, if we use the rule that the Gauss diagram skeleton is not allowed to use virtual crossings.
What kinds of chords do our integrals detect? They detect "semi-virtuals with outer rings".
(Conjectured) Punchline: Relations on Feynman diagrams correspond with relations on chord diagrams. This is just a matter of carefully checking the analogues of the relations we already knew. What makes this work here and not in the original theory is that we have degree 2 chords, the semi-virtuals.
What would be nice is a clean formulation of finite type for virtual doodles yielding chords which are "semi-virtuals with outer rings".

So maybe "virtual doodles" isn't the right name for this $\mathbb{Z}_6$ or maybe we should think of Gauss diagrams as containing "rotation information" $\mathbb{Z}_6$

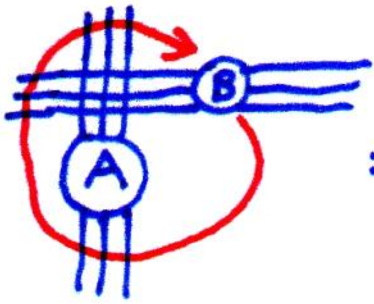




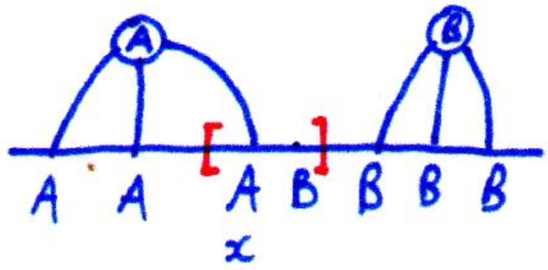


Perhaps a "virtual doodle" is a GD along with a $\frac{1}{4}\mathbb{Z}$-valued "rotation pairing" on $Z_1 \times C_0$ (with correct "stepping").
(nay, homotopy information should be recorded too)

Relations on Chord Diagrams



$\Rightarrow \sum_{x \in A}$



$+ \sum_{x \in B}$ (2NT)