The 17 Worlds of Planar Ants

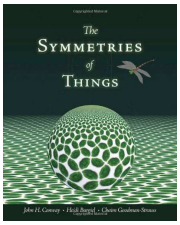
Goal. Get you hooked!

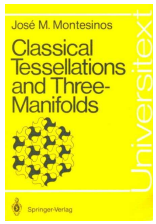
Video, handout, links at ω/

Abstract. Back in early 2000, I got my first digital camera and set out to take pictures of my kids and of symmetric patterns in the plane (ω/Tilings). There are exactly 17 of those, no more, no less. It is an addicting challenge to walk around looking at buildings, brick walls, people's ties, fabrics, what's not, and to try figure out which of the 17 is each one.

- What would history look like if we were living on Venus?
- What do the ants on Lou Kauffman's tie think?

Lou Kauffman's Tie

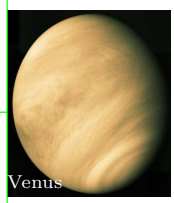


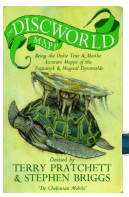


Books.

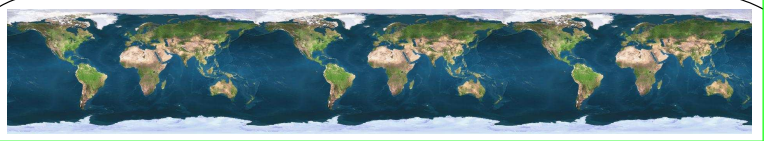
- J. H. Conway, H. Burgiel, and C. Goodman-Strauss, *The Symmetries of Things*, CRC Press, 2008.
- J. M. Montesinos, *Classical Tessellations and Three-Manifolds*, Springer-Verlag, 1987.

The Venus Story



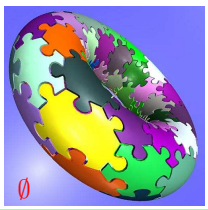


ω/DW



The Renaissance Story

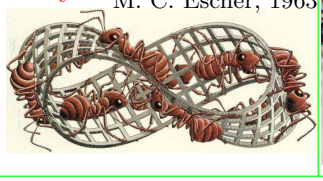
ω/Longtin



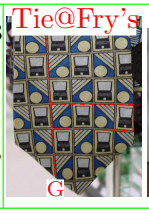
The Lake Merrit Story

The Racha Cafe Story

M. C. Escher, 1963



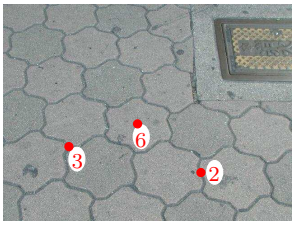
Tie@Fry's

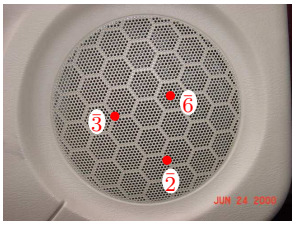


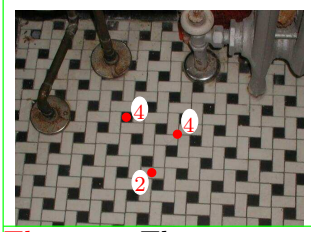
ω/Sanderson

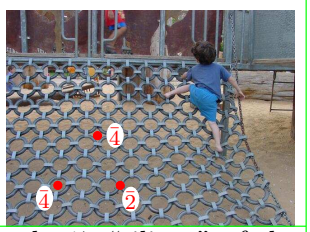
Claim.

Exactly 10 "features" are possible. They are M, G, 2, 3, 4, 6, $\bar{2}$, $\bar{3}$, $\bar{4}$, and $\bar{6}$.









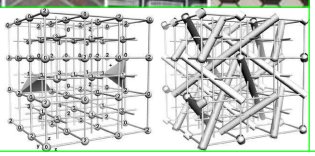
Theorem.

There are exactly 17 "tilings" of the plane: ∅=0, MM=∗∗, MG=∗∘, GG=∘∘, 2222=2222, 333=333, 442=442, 632=632, $\bar{2}\bar{2}\bar{2}\bar{2}$=∗2222, $\bar{3}\bar{3}\bar{3}$=∗333, $\bar{4}\bar{4}\bar{2}$=∗442, $\bar{6}\bar{3}\bar{2}$=∗632, $4\bar{2}$=4∗2, $3\bar{3}$=3∗3, $2\bar{2}\bar{2}$=2∗22, 22M=22∗, 22G=22∘.

18??

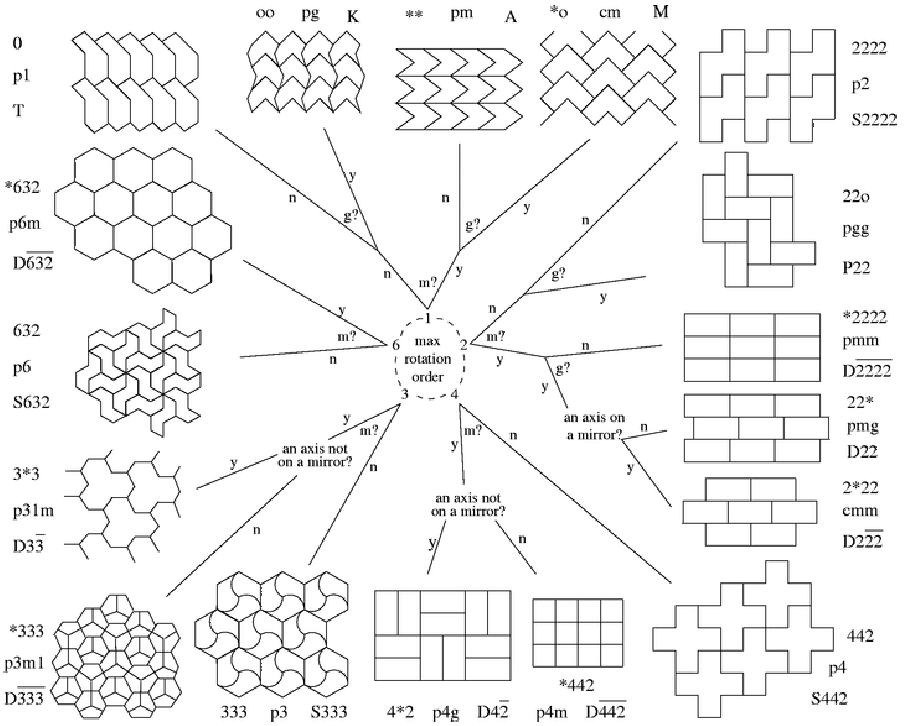
The 230 Worlds of Spacial Monkeys (The 219 worlds of Monkeys that Can't Tell their Left from their Right)

ω/Crys, ω/CFHT

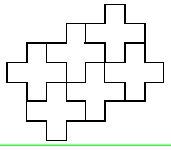


Brian Sanderson's Pattern Recognition Algorithm

Is the maximum rotation order 1,2,3,4 or 6? Is there a mirror (m)? Is there an indecomposable glide reflection (g)? Is there a rotation axis on a mirror? Is there a rotation axis not on a mirror?



Note: Every pattern is identified according to three systems of notation, as in the example below:



442: The Conway-Thurston notation, as used in my tilings page.

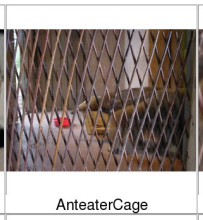
p4: The International Union of Crystallography notation.

S442: The Montesinos notation, as in his book Classical Tesselations and Three Manifolds

Dror Bar-Natan's tilings worksheet: Classify the following pictures according to the following possibilities: 2222, 333, 442, 632, *2222, *333, *442, *632, 4*2, 3*3, 2*22, 22*, **, *o, oo, 22o, and 0 (the pictures come in {context, pattern} pairs).


Alhambra

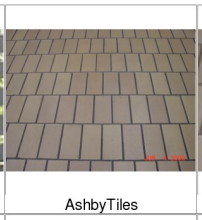

Alhambra_120


AnteaterCage

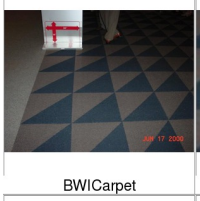

AnteaterCage_120


ArchStreetFence


ArchStreetFence_sm


AshbyTiles


AshbyTiles_120


BWICarpet


BWICarpet_120


BathroomTiles_120


BathroomTiles_640


BedCoverAndAnnie


BedCoverEnlarged


BethElQuilt


BethElQuilt_120


BethElSidewalk


BethElSidewalk_sm


BethlehemRoadTiles


BethlehemRoadTiles_120


BicycleReflector

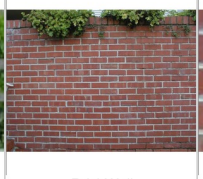

BicycleReflector_120


BrakePedal


BrakePedal_120


BrickPartition

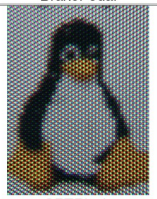

BrickPartition_120

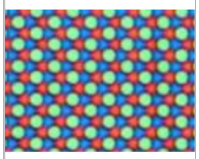

BrickWall


BrickWall_sm


BriennesNecklace


BriennesNecklace_120


CRTPixels


CRTPixels_120


Cactus


Cactus_120


CaesareaMosaic_120


CaesareaMosaic_640


ChildsDrawing


ChildsDrawing_sm


CleaningCloth


CleaningCloth_120

ClimbingNet

ClimbingNet_120

DSC-S70Pixels

DSC-S70Pixels_120

DamascusTable

DamascusTable_120

EowynRieke

EowynRieke_120

ExploratoriumBathroom

ExploratoriumBathroom_sm

ExternalWallTiles

ExternalWallTiles_120

FRACeiling

FRACeiling_120

FlowerBasket

FlowerBasket_sm

FlowersWallpaper

FlowersWallpaper_120

FootstepInSand

FootstepInSand_120

Footstool

Footstool_120

FrontGrill

FrontGrill_sm

FrozenFence

FrozenFence_240

FutureShop_120

FutureShop_640

GarageDoor

GarageDoor_sm

GatedDoorway

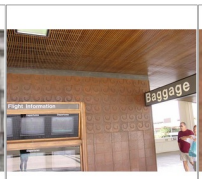
GatedDoorway_sm


GionTiles_120


GionTiles_640


GlassWall

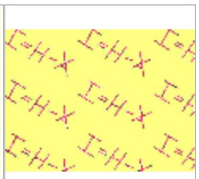

GlassWall_120



HNLWall

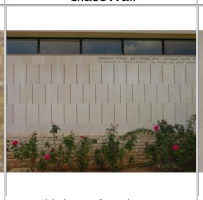

HNLWall_120


HTMLBackground


HTMLBackground_120


HarHaMenuhotWall


HarHaMenuhotWall_120


HebrewAcademy


HebrewAcademy_120


HexGame


HexGame_sm


HotPad


HotPad_120


IncentBurner

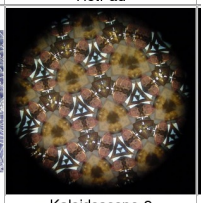

IncentBurner_120

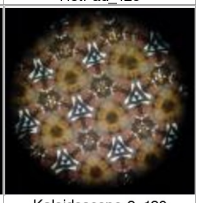

JCCBench

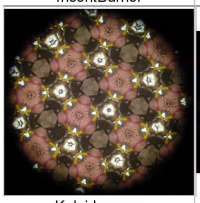

JCCBench_sm

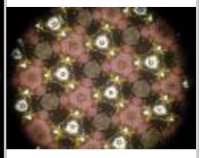

JapanesePlate_120


JapanesePlate_800


Kaleidoscope-2


Kaleidoscope-2_120


Kaleidoscope

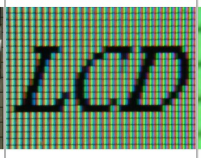

Kaleidoscope_120


KauffmanTie


KauffmansTie_1280


KuramaguchiStation_120

KuramaguchiStation_640

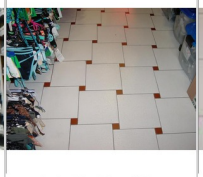


LCDPixels


LCDPixels_120


LakeMerritBuilding


LakeMerritEnlarged


LalushkaFloorTiles


LalushkaFloorTiles_120


Lego

Lego_120


LittleItalyChair


LittleItalyChair_120

MSRIBathroom

MSRIBathroom_sm

MayasTablecloth

MayasTablecloth_120

MetalGrating

MetalGrating_sm

MorrocanTabletop_120

MorrocanTabletop_640

NewGeometrics

NewGeometrics_120

NicholasKZieveShirt

NicholasKZieveShirt_120

NonSlid

NonSlid_120

OldRadio

OldRadio_120

OutsideWall

OutsideWall_sm

OxfordStreetEntryway

OxfordStreetEntryway_sm

PaperTowel

PaperTowel_120

ParkingLotTiles

ParkingLotTiles_120

PentagonalTiles

PentagonalTiles_120

PicnicBasket

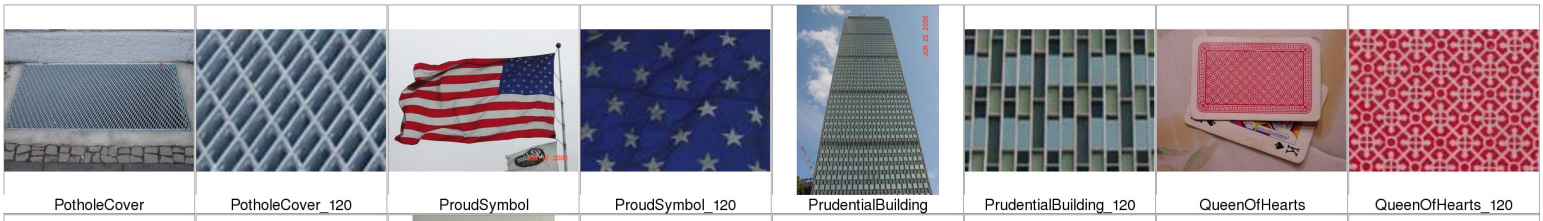
PicnicBasket_120

PizzaUnoFloorTiles

PizzaUnoFloorTiles_120

PlaygroundFence

PlaygroundFence_120


PotholeCover

PotholeCover_120

ProudSymbol

ProudSymbol_120

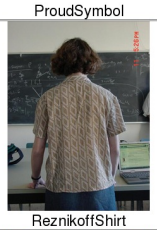
PrudentialBuilding

PrudentialBuilding_120

QueenOfHearts

QueenOfHearts_120

RenaissanceHotel

RenaissanceHotel_120

ReznikoffShirt

ReznikoffShirt_120

SFGlassWindow

SFGlassWindow_sm

SableSpeaker

SableSpeaker_120

SafewayCart

SafewayCart_sm

Shredder

Shredder_120

SidewalkTiles

SidewalkTiles_120

SinkDrainer

SinkDrainer_sm

SisInLawShirt

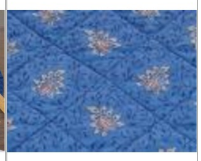
SisInLawShirt_120

SportsFieldFence

SportsFieldFence_120

SugarPacket

SugarPacket_sm

TableMat

TableMat_120

TanHall

TanHall_sm

ThaiWallpaper

ThaiWallpaper_sm


TheStudioBuilding

TheStudioBuilding_sm

TieAtFrys

TieAtFrys_120

TieAtSFO

TieAtSFO_120

TieAtSears

TieAtSears_120

TireThreads

TireThreads_120

ToiletPaper

ToiletPaper_120

UnderKnapp

UnderKnapp_120

WalnutStreetBuilding

WalnutStreetBuilding_sm

WalthamParkTiles

WalthamParkTiles_120

WaterPipes

WaterPipes_120

WineBottles

WineBottles_120

WovenBasket

WovenBasket_sm

WovenBenchBack

WovenBenchBack_120

Yeshua

Yeshua_120

artificial-333-full

artificial-333

This worksheet: http://drorbn.net/AcademicPensieve/2014-08/17Worlds/WorkSheet.pdf
Solutions and more: http://www.math.toronto.edu/~drorbn/Gallery/Symmetry/Tilings/
Talk: http://www.math.toronto.edu/~drorbn/Talks/ClassroomAdventures-1408/