Unitarity of Burau, take II

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$$U_i = \begin{pmatrix} I_i & & & \\ & 1-t & t & \\ & 1 & 0 & \\ & & & I_{n-i-1} \end{pmatrix}, \quad U_i^{-1} = \begin{pmatrix} I_i & & & \\ & 0 & 1 & \\ & \bar{t} & 1-\bar{t} & \\ & & & I_{n-i-1} \end{pmatrix}. \text{ Unitarity: } \bar{U}\Omega_n U^T = \Omega_n \text{ with } \Omega_n = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1-t & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1-t & 1-t & \cdots & 1 \end{pmatrix}$$

$$U \Omega_n^T \bar{U}^T = \Omega_n^T \iff \bar{U}^{-1}\Omega_n = \Omega_n U^T$$

Let $B = $ [a block of 1's].

$$\bar{U}^{-1}\Omega_n = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & t & 1-t & 0 \\ 0 & 0 & 0 & I \end{pmatrix} \begin{pmatrix} \Omega & 0 & 0 & 0 \\ \underline{(1-t)B} & 1 & 0 & 0 \\ \underline{(1-t)B} & 1-t & 1 & 0 \\ (1-t)B & (1-t)B & (1-t)B & \Omega \end{pmatrix}$$

$$= \begin{bmatrix} \Omega & 0 & 0 & 0 \\ (1-t)B & 1-t & 1 & 0 \\ \underline{(1-t)B}_2 & \underline{1-t+t^2}_1 & 1-t & 0 \\ (1-t)B & (1-t)B & (1-t)B & \Omega \end{bmatrix}$$

1: $t+(1-t)^2 = 1-t+t^2$

2: use 2-green equality.

$$\Omega_n U^T = \begin{pmatrix} \Omega & 0 & 0 & 0 \\ (1-t)B & 1 & 0 & 0 \\ (1-t)B & 1-t & 1 & 0 \\ (1-t)B & \underline{(1-t)B} & \underline{(1-t)B} & \Omega \end{pmatrix} \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 1-t & 1 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$$

$$= \begin{bmatrix} \Omega & 0 & 0 & 0 \\ (1-t)B & 1-t & 1 & 0 \\ (1-t)B & \underline{1-t+t^2}_1 & 1-t & 0 \\ (1-t)B & \underline{(1-t)B}_2 & (1-t)B & \Omega \end{bmatrix}$$

June 27 aside:

$$\frac{t_i}{1-t_i} + (1-t_i)1 = \dots$$

$$1(1-t_i) + \frac{1}{1-t_i+0}$$

$$t+(1-t)(1-s) = 1-s+ts$$