Unitarity at 2x2

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$$\begin{pmatrix} 0 & 1 \\ \bar{a} & 1-\bar{a} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1-a & a \\ 1 & 0 \end{pmatrix}$$

At 2x2, I'm looking for $M(a,b)$ s.t.

1. $$\begin{pmatrix} 0 & 1 \\ \bar{a} & 1-\bar{a} \end{pmatrix} M(b,a) = M(a,b) \begin{pmatrix} 1-\bar{a} & 1 \\ \bar{a} & 0 \end{pmatrix}$$

2. $$\begin{pmatrix} 1-b & b \\ 1 & 0 \end{pmatrix} M(b,a) = M(a,b) \begin{pmatrix} 0 & b \\ 1 & 1-b \end{pmatrix}$$

with $M(b,a) = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$, $M(a,b) = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$

1. $$\begin{pmatrix} q_{21} & q_{22} \\ \bar{a}q_{11} + (1-\bar{a})q_{21} & \bar{a}q_{12} + (1-\bar{a})q_{22} \end{pmatrix} = \begin{pmatrix} (1-\bar{a})p_{11} + \bar{a}p_{12} & p_{11} \\ (1-\bar{a})p_{21} + \bar{a}p_{22} & p_{21} \end{pmatrix}$$

2. $$\begin{pmatrix} (1-b)q_{11} + bq_{21} & (1-b)q_{12} + bq_{22} \\ q_{11} & q_{12} \end{pmatrix} = \begin{pmatrix} p_{12} & bp_{11} + (1-b)p_{12} \\ p_{22} & bp_{21} + (1-b)p_{22} \end{pmatrix}$$

$\Longrightarrow$ $q_{22} = p_{11}$ $\quad q_{11} = p_{22}$ $\quad q_{21} = p_{21}$ $\quad q_{12} = p_{12}$

$$p_{21} = (1-\bar{a})p_{11} + \bar{a}p_{12}$$
$$p_{12} = (1-b)p_{22} + bp_{21}$$

~~$p_{21} = \bar{a}p_{12} + (1-\bar{a})p_{11}$~~

Say $p_{21} = p_{12} = 1$, then $p_{11} = 1 = p_{22}$ so $M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\Longrightarrow$ no information.

Say $p_{21} = 1$, $p_{12} = 0$ $\Longrightarrow$ $p_{11} = \dfrac{1}{1-\bar{a}}$ $\quad p_{22} = -\dfrac{b}{1-b}$

$$= \frac{1}{1-b}$$

Yay!! $$M = \begin{pmatrix} \frac{a}{a-1} & 0 \\ 1 & \frac{b}{b-1} \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{a-1} & 0 \\ 1 & 1 + \frac{1}{b-1} \end{pmatrix}$$

Using linearity and substracting $\begin{pmatrix} 1 & 1 \\ & 1 \end{pmatrix}$, we see that $\begin{pmatrix} \frac{1}{a-1} & -1 \\ 0 & \frac{1}{b-1} \end{pmatrix}$ also works, and so does

$$\begin{pmatrix} \frac{1}{1-a} & 1 \\ 0 & \frac{1}{1-b} \end{pmatrix}$$

At $a=b=T$, This reduces to The unitarity of Burau! Does this pattern persist?