Divisibility conditions for Gassner calculus

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$(T_a-1) \mid A_{ab}-\delta_{ab}\sigma_b$?

Gassner calculus. Preserves $C_1 :=$ [col sum = 1] ($\Leftrightarrow$OC) and $C_2 := \boxed{[\forall a\neq b,\ (T_a-1)\mid A_{ab}]}$

$$\left(\begin{array}{c|cc} \nu\omega & c & S \\ \hline c & \beta+\alpha\delta/\nu & \theta+\alpha\epsilon/\nu \\ S & \psi+\delta\phi/\nu & \Xi+\epsilon\phi/\nu \end{array}\right)_{T_a,T_b\to T_c} \xleftarrow[\nu:=1-\gamma]{m_c^{ba}} \begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\beta]{m_c^{ab}} \left(\begin{array}{c|cc} \mu\omega & c & S \\ \hline c & \gamma+\alpha\delta/\mu & \epsilon+\delta\theta/\mu \\ S & \phi+\alpha\psi/\mu & \Xi+\theta\psi/\mu \end{array}\right)_{T_a,T_b\to T_c} \qquad R^{\pm}_{ab} \underset{\gamma}{=} \begin{array}{c|cc} 1 & a & b \\ \hline a & 1 & 1-T_a^{\pm1} \\ b & 0 & T_a^{\pm1} \end{array}$$

$$\begin{array}{c|cc} \omega & a & S \\ \hline a & \alpha & \theta \\ S & \phi & \Xi \end{array} \xrightarrow[\substack{\mu:=T_a-1\\ \nu:=\alpha-\sigma_a}]{q\Delta^a_{bc}} \left(\begin{array}{c|ccc} \omega & b & c & S \\ \hline b & (\sigma_a-\alpha T_a-\nu T_c)/\mu & (T_b-1)T_c\nu/\mu & (T_b-1)T_c\theta/\mu \\ c & (T_c-1)\nu/\mu & (\alpha-\sigma_a T_a-\nu T_c)/\mu & (T_c-1)\theta/\mu \\ S & \phi & \phi & \Xi \end{array}\right)_{T_a\mapsto T_bT_c}$$

Satisfies: ✓ $R^+_{13}/\!\!/q\Delta^1_{12} = R^+_{23}\#R^+_{13}$.
✓ $R^-_{13}/\!\!/q\Delta^1_{12} = R^-_{13}\#R^-_{23}$.
✓ $q\Delta^a_{a_1a_2}/\!\!/q\Delta^b_{b_1b_2}/\!\!/m^{a_1b_1}_{c_1}/\!\!/m^{a_2b_2}_{c_2}$
$= m^{ab}_c/\!\!/q\Delta^c_{c_1c_2}$.

$$\begin{array}{c|cc} \omega & a & S \\ \hline a & \alpha & \theta \\ S & \phi & \Xi \end{array} \xrightarrow{dS^a} \left(\begin{array}{c|cc} \alpha\omega/\sigma_a & a & S \\ \hline a & 1/\alpha & \theta/\alpha \\ S & -\phi/\alpha & (\alpha\Xi-\phi\theta)/\alpha \end{array}\right)_{T_a\to T_a^{-1}}$$

Satisfies: ✓ $R^{\pm}_{12}/\!\!/dS^{1\text{ or }2} = R^{\mp}_{12}$. ✓ $dm^{ab}_c/\!\!/dS^c = dS^a/\!\!/dS^b/\!\!/dm^{ba}_c$.
✓ $dS^a/\!\!/dS^a = I$. ✓ $q\Delta^a_{bc}/\!\!/dS^b/\!\!/dS^c = dS^a/\!\!/q\Delta^a_{cb}$.
✓ Assuming C_2, $d\eta^a/\!\!/d\epsilon_a = q\Delta^a_{bc}/\!\!/dS^c/\!\!/dm^{bc}_a$ (also 3 variants).

The map (tangle $T\mapsto$ matrix A) is anti-multiplicative.

The MVA mod units: $L\mapsto(\omega,A)\mapsto\omega\det{}'(A-I)/(1-T')$ ✓

At $T_*=1$: $W=1$, $\sigma_*=1$ & $A=I$. } add.

$$(\sigma_a-\alpha T_a-(\alpha-\sigma_a)T_c)/(T_a-1) - \sigma_a =$$

$$\frac{\sigma_a-\alpha T_bT_c-\alpha T_c+\sigma_a T_c+\sigma_a-T_bT_c\sigma_a}{T_bT_c-1}$$

$$=\frac{\sigma_a-\alpha T_bT_c-\alpha+\alpha-\alpha T_c+\sigma_a T_c+\sigma_a-T_bT_c\sigma_a}{T_bT_c-1}$$

$$=\frac{\sigma_a-\alpha}{T_bT_c-1}-\alpha+\frac{\sigma_a-\alpha}{T_bT_c-1}T_c-\sigma_a$$

$$=(1+T_c)\frac{\sigma_a-\alpha}{T_bT_c-1}-\sigma_a-\alpha$$

In DivisibilityCondition.nb:

Using $(T_x T_y - 1)|(\alpha\text{-}\sigma a)$ get $\sigma a = \alpha - q\,(T_x T_y - 1)$ and then:

(Alt) In[1]:= $\text{Simplify}\left[\dfrac{-\sigma a + \alpha\, Ta + (-\alpha + \sigma a)\, T_y}{-1 + Ta} - \sigma a\ /.\ \{Ta \to T_x T_y,\ \sigma a \to \alpha - q\,(T_x T_y - 1)\}\right]$

(Alt) Out[1]= $q\,(-1 + T_x)\,T_y$

(Alt) In[2]:= $\text{Simplify}\left[\dfrac{-\alpha + \sigma a\, Ta + (\alpha - \sigma a)\, T_y}{-1 + Ta} - \sigma a\ /.\ \{Ta \to T_x T_y,\ \sigma a \to \alpha - q\,(T_x T_y - 1)\}\right]$

(Alt) Out[2]= $q\,(-1 + T_y)$