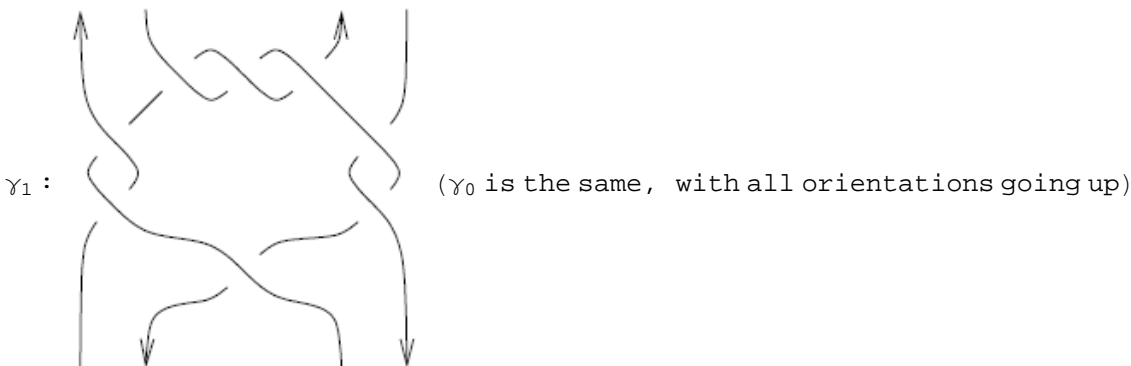


Pensieve Header: An attempt on the ribbon property using Γ -calculus; continues RibbonPropertyV1.
 Success at last!!!

```
dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-06/"];
<< KnotTheory`
<< MetaCalculi/MetaCalculi-Program.m
Format[ $\alpha_{a,b}$ , StandardForm] := Interpretation[ $\alpha_{10\ a+b}$ ,  $\alpha_{ab}$ ];
Format[ $\beta_{a,b}$ , StandardForm] := Interpretation[ $\beta_{10\ a+b}$ ,  $\beta_{ab}$ ];

Loading KnotTheory` version of April 3, 2014, 16:23:56.0784.
Read more at http://katlas.org/wiki/KnotTheory.
```



$$\{n = 4; \gamma_0 = \Gamma[\omega, \sum_{a=0}^n h_a \sigma_a, \sum_{a=1}^n \sum_{b=1}^n t_a h_b \alpha_{ab}], \gamma_1 = \gamma_0 // ds[2] // ds[4]\}$$

$$\left(\begin{array}{c|cccccc} \omega & S_1 & S_2 & S_3 & S_4 & \\ \hline S_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \\ S_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \\ S_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \\ S_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \end{array} \right), \left(\begin{array}{c|cccccc} & & & & & \\ \hline S_1 & \frac{\omega(\alpha_{24}\alpha_{42}-\alpha_{22}\alpha_{44})}{\sigma_2\sigma_4} & & & & \\ S_2 & \frac{\alpha_{14}\alpha_{22}\alpha_{41}-\alpha_{12}\alpha_{24}\alpha_{41}-\alpha_{14}\alpha_{21}\alpha_{42}+\alpha_{11}\alpha_{24}\alpha_{42}+\alpha_{12}\alpha_{21}\alpha_{44}-\alpha_{11}\alpha_{22}\alpha_{44}}{\alpha_{24}\alpha_{42}-\alpha_{22}\alpha_{44}} & & & & \\ S_3 & \frac{\alpha_{24}\alpha_{41}-\alpha_{21}\alpha_{44}}{\alpha_{24}\alpha_{42}-\alpha_{22}\alpha_{44}} & & & & \\ S_4 & \frac{-\alpha_{24}\alpha_{32}\alpha_{41}+\alpha_{22}\alpha_{34}\alpha_{41}+\alpha_{24}\alpha_{31}\alpha_{42}-\alpha_{21}\alpha_{34}\alpha_{42}-\alpha_{22}\alpha_{31}\alpha_{44}+\alpha_{21}\alpha_{32}\alpha_{44}}{\alpha_{24}\alpha_{42}-\alpha_{22}\alpha_{44}} & & & & \\ \Sigma & \frac{\alpha_{22}\alpha_{41}-\alpha_{21}\alpha_{42}}{-\alpha_{24}\alpha_{42}+\alpha_{22}\alpha_{44}} & & & & \end{array} \right), \left(\begin{array}{c} S \\ \hline \alpha_{14}\alpha_4 \\ \alpha_{24}\alpha_4 \\ \alpha \\ -\alpha_{24}\alpha_{42} \\ -\alpha_{34}\alpha_4 \\ \alpha_{24}\alpha_4 \\ \alpha \\ \alpha_{24}\alpha_{42} \\ \vdots \\ - \\ \Sigma \end{array} \right)$$

```
{Ov = Xp[o1, 1] Xp[o2, 2] Xp[o3, 3] Xp[o4, 4] // \Gamma // dm[o1, o2, o] // dm[o, o3, o] // dm[o, o4, o],
t1 = Ov ** (\gamma_0 * \Gamma[\epsilon[o]]), t2 = (\gamma_0 * \Gamma[\epsilon[o]]) ** Ov, ocond = Simplify[t1 == t2]}
```

$$\left(\begin{array}{c|cccccc} 1 & S_1 & S_2 & S_3 & S_4 & S_o \\ \hline S_1 & T_o & 0 & 0 & 0 & 0 \\ S_2 & 0 & T_o & 0 & 0 & 0 \\ S_3 & 0 & 0 & T_o & 0 & 0 \\ S_4 & 0 & 0 & 0 & T_o & 0 \\ S_o & 1-T_o & 1-T_o & 1-T_o & 1-T_o & 1 \\ \Sigma & T_o & T_o & T_o & T_o & 1 \end{array} \right), \left(\begin{array}{c|cccccc} \omega & S_1 & S_2 & S_3 & S_4 & S_o \\ \hline S_1 & T_o \alpha_{11} & T_o \alpha_{12} & T_o \alpha_{13} & T_o \alpha_{14} & 0 \\ S_2 & T_o \alpha_{21} & T_o \alpha_{22} & T_o \alpha_{23} & T_o \alpha_{24} & 0 \\ S_3 & T_o \alpha_{31} & T_o \alpha_{32} & T_o \alpha_{33} & T_o \alpha_{34} & 0 \\ S_4 & T_o \alpha_{41} & T_o \alpha_{42} & T_o \alpha_{43} & T_o \alpha_{44} & 0 \\ S_o & 1-T_o & 1-T_o & 1-T_o & 1-T_o & 1 \\ \Sigma & T_o \sigma_1 & T_o \sigma_2 & T_o \sigma_3 & T_o \sigma_4 & 1 \end{array} \right), \left(\begin{array}{c} \omega \\ \hline S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_o \\ \Sigma \end{array} \right), \left(\begin{array}{c} T_o \\ T_o \\ T_o \\ T_o \\ T_o \\ -(-1+T_o)(\alpha_{11} \\ T_c \end{array} \right)$$

$$\left. \begin{aligned} (-1+T_o)(-1+\alpha_{11}+\alpha_{21}+\alpha_{31}+\alpha_{41}) &= 0 \ \&\& \ (-1+T_o)(-1+\alpha_{12}+\alpha_{22}+\alpha_{32}+\alpha_{42}) = 0 \ \&\& \\ (-1+T_o)(-1+\alpha_{13}+\alpha_{23}+\alpha_{33}+\alpha_{43}) &= 0 \ \&\& \ (-1+T_o)(-1+\alpha_{14}+\alpha_{24}+\alpha_{34}+\alpha_{44}) = 0 \end{aligned} \right\}$$

```
ocond = FullSimplify[ocond /. {T0 -> 0}]
```

$$\alpha_{11} + \alpha_{21} + \alpha_{31} + \alpha_{41} == 1 \ \&\& \ \alpha_{12} + \alpha_{22} + \alpha_{32} + \alpha_{42} == 1 \ \&\& \ \alpha_{13} + \alpha_{23} + \alpha_{33} + \alpha_{43} == 1 \ \&\& \ \alpha_{14} + \alpha_{24} + \alpha_{34} + \alpha_{44} == 1$$

```
{U = Xm[1, u1] Xm[2, u2] Xm[3, u3] Xm[4, u4] // r // dm[u1, u2, u] // dm[u, u3, u] //
  dm[u, u4, u],
  t1 = U** (gamma0 * r[e[u]]), t2 = (gamma0 * r[e[u]]) ** U, ucond = FullSimplify[t1 == t2]}
```

$$\left(\begin{array}{cccccc} 1 & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & 1 & 0 & 0 & 0 & \frac{-1+T_1}{T_1} \\ s_2 & 0 & 1 & 0 & 0 & \frac{-1+T_2}{T_1 T_2} \\ s_3 & 0 & 0 & 1 & 0 & \frac{-1+T_3}{T_1 T_2 T_3} \\ s_4 & 0 & 0 & 0 & 1 & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & 1 & 1 & 1 & 1 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\left(\begin{array}{cccccc} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-T_2 T_3 T_4 \alpha_{11} + T_1 T_2 T_3 T_4 \alpha_{11} - T_3 T_4 \alpha_{12} + T_2 T_3 T_4 \alpha_{12} - T_4 \alpha_{13} + T_3 T_4 \alpha_{13} - \alpha_{14} + T_4 \alpha_{14}}{T_1 T_2 T_3 T_4} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-T_2 T_3 T_4 \alpha_{21} + T_1 T_2 T_3 T_4 \alpha_{21} - T_3 T_4 \alpha_{22} + T_2 T_3 T_4 \alpha_{22} - T_4 \alpha_{23} + T_3 T_4 \alpha_{23} - \alpha_{24} + T_4 \alpha_{24}}{T_1 T_2 T_3 T_4} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-T_2 T_3 T_4 \alpha_{31} + T_1 T_2 T_3 T_4 \alpha_{31} - T_3 T_4 \alpha_{32} + T_2 T_3 T_4 \alpha_{32} - T_4 \alpha_{33} + T_3 T_4 \alpha_{33} - \alpha_{34} + T_4 \alpha_{34}}{T_1 T_2 T_3 T_4} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-T_2 T_3 T_4 \alpha_{41} + T_1 T_2 T_3 T_4 \alpha_{41} - T_3 T_4 \alpha_{42} + T_2 T_3 T_4 \alpha_{42} - T_4 \alpha_{43} + T_3 T_4 \alpha_{43} - \alpha_{44} + T_4 \alpha_{44}}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\left(\begin{array}{cccccc} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-1+T_1}{T_1} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-1+T_2}{T_1 T_2} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-1+T_3}{T_1 T_2 T_3} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\left. \begin{aligned} & \frac{1}{T_1 T_2 T_3 T_4} (T_4 (T_3 ((-1 + T_1) T_2 (-1 + \alpha_{11}) + (-1 + T_2) \alpha_{12}) + (-1 + T_3) \alpha_{13}) + (-1 + T_4) \alpha_{14}) == 0 \ \&\& \\ & \frac{1}{T_1 T_2 T_3 T_4} (T_4 (-\alpha_{23} + T_3 (1 - \alpha_{22} + T_2 (-1 + (-1 + T_1) \alpha_{21} + \alpha_{22}) + \alpha_{23})) + (-1 + T_4) \alpha_{24}) == 0 \ \&\& \\ & \frac{1}{T_1 T_2 T_3 T_4} (-\alpha_{34} + T_4 (1 - \alpha_{33} + T_3 (-1 - \alpha_{32} + T_2 ((-1 + T_1) \alpha_{31} + \alpha_{32}) + \alpha_{33}) + \alpha_{34})) == 0 \ \&\& \\ & \frac{1}{T_1 T_2 T_3 T_4} (1 - \alpha_{44} + T_4 (-1 - \alpha_{43} + T_3 (-\alpha_{42} + T_2 ((-1 + T_1) \alpha_{41} + \alpha_{42}) + \alpha_{43}) + \alpha_{44})) == 0 \end{aligned} \right\}$$

```
{cert = γ1 // dm[1, 2, 1] // dm[3, 4, 2],
 eqns = (ε[1] ε[2] // Γ) == (cert /. σ_ → 1) // Simplify}
```

$$\left\{ \begin{array}{l} \frac{\omega (\alpha_{14} \alpha_{32} + \alpha_{24} \alpha_{32} - \alpha_{12} \alpha_{34} - \alpha_{22} \alpha_{34} + \alpha_{14} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{12} \alpha_{44} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} \\ S_1 \\ S_2 \\ \Sigma \end{array} \right. \begin{array}{l} S_1 \\ \frac{\alpha_{14} \alpha_{31} + \alpha_{24} \alpha_{31} - \alpha_{11} \alpha_{34} - \alpha_{21} \alpha_{34} + \alpha_{14} \alpha_{41} + \alpha_{24} \alpha_{41} - \alpha_{11} \alpha_{44} - \alpha_{21} \alpha_{44}}{\alpha_{14} \alpha_{32} + \alpha_{24} \alpha_{32} - \alpha_{12} \alpha_{34} - \alpha_{22} \alpha_{34} + \alpha_{14} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{12} \alpha_{44} - \alpha_{22} \alpha_{44}} \frac{\alpha_{14}}{\alpha_{14}} \\ \frac{\alpha_{12} \alpha_{31} + \alpha_{22} \alpha_{31} - \alpha_{11} \alpha_{32} - \alpha_{21} \alpha_{32} + \alpha_{12} \alpha_{41} + \alpha_{22} \alpha_{41} - \alpha_{11} \alpha_{42} - \alpha_{21} \alpha_{42}}{-\alpha_{14} \alpha_{32} - \alpha_{24} \alpha_{32} + \alpha_{12} \alpha_{34} + \alpha_{22} \alpha_{34} - \alpha_{14} \alpha_{42} - \alpha_{24} \alpha_{42} + \alpha_{12} \alpha_{44} + \alpha_{22} \alpha_{44}} \frac{\alpha_{13}}{\alpha_{14}} \\ \frac{\sigma_1}{\sigma_2} \end{array}$$

$$\left. \begin{array}{l} \frac{\alpha_{14} (\alpha_{31} + \alpha_{41}) + \alpha_{24} (\alpha_{31} + \alpha_{41}) - (\alpha_{11} + \alpha_{21}) (\alpha_{34} + \alpha_{44})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 1 \ \&\& \\ \frac{\alpha_{14} (\alpha_{33} + \alpha_{43}) + \alpha_{24} (\alpha_{33} + \alpha_{43}) - (\alpha_{13} + \alpha_{23}) (\alpha_{34} + \alpha_{44})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 0 \ \&\& \\ \frac{\alpha_{12} (\alpha_{31} + \alpha_{41}) + \alpha_{22} (\alpha_{31} + \alpha_{41}) - (\alpha_{11} + \alpha_{21}) (\alpha_{32} + \alpha_{42})}{-\alpha_{14} (\alpha_{32} + \alpha_{42}) - \alpha_{24} (\alpha_{32} + \alpha_{42}) + (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 0 \ \&\& \\ \frac{\alpha_{13} (\alpha_{32} + \alpha_{42}) + \alpha_{23} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{33} + \alpha_{43})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 1 \end{array} \right\}$$

```
γ1 // dm[2, 1, 1] // dm[4, 3, 3]
```

$$\left\{ \begin{array}{l} \frac{\omega (\alpha_{23} \alpha_{41} - \alpha_{24} \alpha_{41} - \alpha_{23} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{21} \alpha_{43} + \alpha_{22} \alpha_{43} + \alpha_{21} \alpha_{44} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} \\ S_1 \\ S_3 \\ \Sigma \end{array} \right. \begin{array}{l} S_1 \\ - \frac{\alpha_{13} \alpha_{41} - \alpha_{14} \alpha_{41} - \alpha_{13} \alpha_{42} + \alpha_{14} \alpha_{42} - \alpha_{11} \alpha_{43} + \alpha_{12} \alpha_{43} + \alpha_{11} \alpha_{44} - \alpha_{12} \alpha_{44}}{\alpha_{23} \alpha_{41} - \alpha_{24} \alpha_{41} - \alpha_{23} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{21} \alpha_{43} + \alpha_{22} \alpha_{43} + \alpha_{21} \alpha_{44} - \alpha_{22} \alpha_{44}} - \frac{\alpha}{\alpha} \\ - \frac{\alpha_{33} \alpha_{41} - \alpha_{34} \alpha_{41} - \alpha_{33} \alpha_{42} + \alpha_{34} \alpha_{42} - \alpha_{31} \alpha_{43} + \alpha_{32} \alpha_{43} + \alpha_{31} \alpha_{44} - \alpha_{32} \alpha_{44}}{\alpha_{23} \alpha_{41} - \alpha_{24} \alpha_{41} - \alpha_{23} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{21} \alpha_{43} + \alpha_{22} \alpha_{43} + \alpha_{21} \alpha_{44} - \alpha_{22} \alpha_{44}} - \frac{\alpha}{\alpha} \\ \frac{\sigma_1}{\sigma_2} \end{array}$$

```
alex = FullSimplify[(γ1 // dm[4, 3, 3] // dm[3, 2, 2] // dm[2, 1, 1])@ω]
```

$$\frac{1}{\sigma_2 \sigma_4} \omega (- (\alpha_{23} - \alpha_{24} + \alpha_{33} - \alpha_{34}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22} + \alpha_{31} - \alpha_{32}) (\alpha_{43} - \alpha_{44}))$$

```
{FullSimplify[alex, eqns], FullSimplify[cert@ω],
 FullSimplify[alex, eqns && ucond && ocond]}
```

$$\left\{ \begin{array}{l} \frac{\omega (- (\alpha_{23} - \alpha_{24}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22}) (\alpha_{43} - \alpha_{44}))}{\sigma_2 \sigma_4} \\ \frac{\omega (- (\alpha_{14} + \alpha_{24}) (\alpha_{32} + \alpha_{42}) + (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44}))}{\sigma_2 \sigma_4} \\ \frac{\omega (- (\alpha_{23} - \alpha_{24}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22}) (\alpha_{43} - \alpha_{44}))}{\sigma_2 \sigma_4} \end{array} \right\}$$

```
Simplify[ $\frac{alex}{cert@ω}$ ]
```

$$- (- (\alpha_{23} - \alpha_{24} + \alpha_{33} - \alpha_{34}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22} + \alpha_{31} - \alpha_{32}) (\alpha_{43} - \alpha_{44})) / (\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44}))$$

Unitarity of γ_0

```
 $\Omega[n] := \text{Table}[\text{Which}[i < j, 0, i == j, \frac{1}{1 - T_i}, i > j, 1], \{i, n\}, \{j, n\}];$ 
```

```
 $\Omega[4] // \text{MatrixForm}$ 
```

$$\begin{pmatrix} \frac{1}{1-T_1} & 0 & 0 & 0 \\ 1 & \frac{1}{1-T_2} & 0 & 0 \\ 1 & 1 & \frac{1}{1-T_3} & 0 \\ 1 & 1 & 1 & \frac{1}{1-T_4} \end{pmatrix}$$

```
 $\text{Transpose}[\gamma_0[A]].\Omega[4].(\gamma_0[A] /. \alpha \rightarrow \beta) // \text{MatrixForm}$ 
```

$$\begin{pmatrix} \left(\frac{\alpha_{11}}{1-T_1} + \alpha_{21} + \alpha_{31} + \alpha_{41}\right) \beta_{11} + \left(\frac{\alpha_{21}}{1-T_2} + \alpha_{31} + \alpha_{41}\right) \beta_{21} + \left(\frac{\alpha_{31}}{1-T_3} + \alpha_{41}\right) \beta_{31} + \frac{\alpha_{41} \beta_{41}}{1-T_4} & \left(\frac{\alpha_{11}}{1-T_1} + \alpha_{21} + \alpha_{31} + \alpha_{41}\right) \beta_{12} \\ \left(\frac{\alpha_{12}}{1-T_1} + \alpha_{22} + \alpha_{32} + \alpha_{42}\right) \beta_{11} + \left(\frac{\alpha_{22}}{1-T_2} + \alpha_{32} + \alpha_{42}\right) \beta_{21} + \left(\frac{\alpha_{32}}{1-T_3} + \alpha_{42}\right) \beta_{31} + \frac{\alpha_{42} \beta_{41}}{1-T_4} & \left(\frac{\alpha_{12}}{1-T_1} + \alpha_{22} + \alpha_{32} + \alpha_{42}\right) \beta_{12} \\ \left(\frac{\alpha_{13}}{1-T_1} + \alpha_{23} + \alpha_{33} + \alpha_{43}\right) \beta_{11} + \left(\frac{\alpha_{23}}{1-T_2} + \alpha_{33} + \alpha_{43}\right) \beta_{21} + \left(\frac{\alpha_{33}}{1-T_3} + \alpha_{43}\right) \beta_{31} + \frac{\alpha_{43} \beta_{41}}{1-T_4} & \left(\frac{\alpha_{13}}{1-T_1} + \alpha_{23} + \alpha_{33} + \alpha_{43}\right) \beta_{12} \\ \left(\frac{\alpha_{14}}{1-T_1} + \alpha_{24} + \alpha_{34} + \alpha_{44}\right) \beta_{11} + \left(\frac{\alpha_{24}}{1-T_2} + \alpha_{34} + \alpha_{44}\right) \beta_{21} + \left(\frac{\alpha_{34}}{1-T_3} + \alpha_{44}\right) \beta_{31} + \frac{\alpha_{44} \beta_{41}}{1-T_4} & \left(\frac{\alpha_{14}}{1-T_1} + \alpha_{24} + \alpha_{34} + \alpha_{44}\right) \beta_{12} \end{pmatrix}$$

```

unital = Simplify[And@@Thread[
  Flatten[Transpose[γ0[A]], Ωc[4].(γ0[A] /. α → β)] == Flatten[Ωc[4]]
]]

```

$$\begin{aligned}
 & \frac{1}{-1 + T_1} + \left(\frac{\alpha_{11}}{1 - T_1} + \alpha_{21} + \alpha_{31} + \alpha_{41} \right) \beta_{11} + \left(\frac{\alpha_{21}}{1 - T_2} + \alpha_{31} + \alpha_{41} \right) \beta_{21} + \left(\frac{\alpha_{31}}{1 - T_3} + \alpha_{41} \right) \beta_{31} + \frac{\alpha_{41} \beta_{41}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{11}}{1 - T_1} + \alpha_{21} + \alpha_{31} + \alpha_{41} \right) \beta_{12} + \left(\frac{\alpha_{21}}{1 - T_2} + \alpha_{31} + \alpha_{41} \right) \beta_{22} + \left(\frac{\alpha_{31}}{1 - T_3} + \alpha_{41} \right) \beta_{32} + \frac{\alpha_{41} \beta_{42}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{11}}{1 - T_1} + \alpha_{21} + \alpha_{31} + \alpha_{41} \right) \beta_{13} + \left(\frac{\alpha_{21}}{1 - T_2} + \alpha_{31} + \alpha_{41} \right) \beta_{23} + \left(\frac{\alpha_{31}}{1 - T_3} + \alpha_{41} \right) \beta_{33} + \frac{\alpha_{41} \beta_{43}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{11}}{1 - T_1} + \alpha_{21} + \alpha_{31} + \alpha_{41} \right) \beta_{14} + \left(\frac{\alpha_{21}}{1 - T_2} + \alpha_{31} + \alpha_{41} \right) \beta_{24} + \left(\frac{\alpha_{31}}{1 - T_3} + \alpha_{41} \right) \beta_{34} + \frac{\alpha_{41} \beta_{44}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{12}}{1 - T_1} + \alpha_{22} + \alpha_{32} + \alpha_{42} \right) \beta_{11} + \left(\frac{\alpha_{22}}{1 - T_2} + \alpha_{32} + \alpha_{42} \right) \beta_{21} + \left(\frac{\alpha_{32}}{1 - T_3} + \alpha_{42} \right) \beta_{31} + \frac{\alpha_{42} \beta_{41}}{1 - T_4} = 1 \ \&\& \\
 & \frac{1}{-1 + T_2} + \left(\frac{\alpha_{12}}{1 - T_1} + \alpha_{22} + \alpha_{32} + \alpha_{42} \right) \beta_{12} + \left(\frac{\alpha_{22}}{1 - T_2} + \alpha_{32} + \alpha_{42} \right) \beta_{22} + \left(\frac{\alpha_{32}}{1 - T_3} + \alpha_{42} \right) \beta_{32} + \frac{\alpha_{42} \beta_{42}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{12}}{1 - T_1} + \alpha_{22} + \alpha_{32} + \alpha_{42} \right) \beta_{13} + \left(\frac{\alpha_{22}}{1 - T_2} + \alpha_{32} + \alpha_{42} \right) \beta_{23} + \left(\frac{\alpha_{32}}{1 - T_3} + \alpha_{42} \right) \beta_{33} + \frac{\alpha_{42} \beta_{43}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{12}}{1 - T_1} + \alpha_{22} + \alpha_{32} + \alpha_{42} \right) \beta_{14} + \left(\frac{\alpha_{22}}{1 - T_2} + \alpha_{32} + \alpha_{42} \right) \beta_{24} + \left(\frac{\alpha_{32}}{1 - T_3} + \alpha_{42} \right) \beta_{34} + \frac{\alpha_{42} \beta_{44}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{13}}{1 - T_1} + \alpha_{23} + \alpha_{33} + \alpha_{43} \right) \beta_{11} + \left(\frac{\alpha_{23}}{1 - T_2} + \alpha_{33} + \alpha_{43} \right) \beta_{21} + \left(\frac{\alpha_{33}}{1 - T_3} + \alpha_{43} \right) \beta_{31} + \frac{\alpha_{43} \beta_{41}}{1 - T_4} = 1 \ \&\& \\
 & \left(\frac{\alpha_{13}}{1 - T_1} + \alpha_{23} + \alpha_{33} + \alpha_{43} \right) \beta_{12} + \left(\frac{\alpha_{23}}{1 - T_2} + \alpha_{33} + \alpha_{43} \right) \beta_{22} + \left(\frac{\alpha_{33}}{1 - T_3} + \alpha_{43} \right) \beta_{32} + \frac{\alpha_{43} \beta_{42}}{1 - T_4} = 1 \ \&\& \\
 & \frac{1}{-1 + T_3} + \left(\frac{\alpha_{13}}{1 - T_1} + \alpha_{23} + \alpha_{33} + \alpha_{43} \right) \beta_{13} + \left(\frac{\alpha_{23}}{1 - T_2} + \alpha_{33} + \alpha_{43} \right) \beta_{23} + \left(\frac{\alpha_{33}}{1 - T_3} + \alpha_{43} \right) \beta_{33} + \frac{\alpha_{43} \beta_{43}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{13}}{1 - T_1} + \alpha_{23} + \alpha_{33} + \alpha_{43} \right) \beta_{14} + \left(\frac{\alpha_{23}}{1 - T_2} + \alpha_{33} + \alpha_{43} \right) \beta_{24} + \left(\frac{\alpha_{33}}{1 - T_3} + \alpha_{43} \right) \beta_{34} + \frac{\alpha_{43} \beta_{44}}{1 - T_4} = 0 \ \&\& \\
 & \left(\frac{\alpha_{14}}{1 - T_1} + \alpha_{24} + \alpha_{34} + \alpha_{44} \right) \beta_{11} + \left(\frac{\alpha_{24}}{1 - T_2} + \alpha_{34} + \alpha_{44} \right) \beta_{21} + \left(\frac{\alpha_{34}}{1 - T_3} + \alpha_{44} \right) \beta_{31} + \frac{\alpha_{44} \beta_{41}}{1 - T_4} = 1 \ \&\& \\
 & \left(\frac{\alpha_{14}}{1 - T_1} + \alpha_{24} + \alpha_{34} + \alpha_{44} \right) \beta_{12} + \left(\frac{\alpha_{24}}{1 - T_2} + \alpha_{34} + \alpha_{44} \right) \beta_{22} + \left(\frac{\alpha_{34}}{1 - T_3} + \alpha_{44} \right) \beta_{32} + \frac{\alpha_{44} \beta_{42}}{1 - T_4} = 1 \ \&\& \\
 & \left(\frac{\alpha_{14}}{1 - T_1} + \alpha_{24} + \alpha_{34} + \alpha_{44} \right) \beta_{13} + \left(\frac{\alpha_{24}}{1 - T_2} + \alpha_{34} + \alpha_{44} \right) \beta_{23} + \left(\frac{\alpha_{34}}{1 - T_3} + \alpha_{44} \right) \beta_{33} + \frac{\alpha_{44} \beta_{43}}{1 - T_4} = 1 \ \&\& \\
 & \frac{1}{-1 + T_4} + \left(\frac{\alpha_{14}}{1 - T_1} + \alpha_{24} + \alpha_{34} + \alpha_{44} \right) \beta_{14} + \left(\frac{\alpha_{24}}{1 - T_2} + \alpha_{34} + \alpha_{44} \right) \beta_{24} + \left(\frac{\alpha_{34}}{1 - T_3} + \alpha_{44} \right) \beta_{34} + \frac{\alpha_{44} \beta_{44}}{1 - T_4} = 0
 \end{aligned}$$

unita2 = Simplify[unital /. {T_{1|3} → T, T_{2|4} → 1/T}]

$$\begin{aligned} & \frac{1}{-1+T} + \left(\frac{\alpha_{11}}{1-T} + \alpha_{21} + \alpha_{31} + \alpha_{41}\right) \beta_{11} + \left(\frac{T\alpha_{21}}{-1+T} + \alpha_{31} + \alpha_{41}\right) \beta_{21} + \left(\frac{\alpha_{31}}{1-T} + \alpha_{41}\right) \beta_{31} + \frac{T\alpha_{41}\beta_{41}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{11}}{1-T} + \alpha_{21} + \alpha_{31} + \alpha_{41}\right) \beta_{12} + \left(\frac{T\alpha_{21}}{-1+T} + \alpha_{31} + \alpha_{41}\right) \beta_{22} + \left(\frac{\alpha_{31}}{1-T} + \alpha_{41}\right) \beta_{32} + \frac{T\alpha_{41}\beta_{42}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{11}}{1-T} + \alpha_{21} + \alpha_{31} + \alpha_{41}\right) \beta_{13} + \left(\frac{T\alpha_{21}}{-1+T} + \alpha_{31} + \alpha_{41}\right) \beta_{23} + \left(\frac{\alpha_{31}}{1-T} + \alpha_{41}\right) \beta_{33} + \frac{T\alpha_{41}\beta_{43}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{11}}{1-T} + \alpha_{21} + \alpha_{31} + \alpha_{41}\right) \beta_{14} + \left(\frac{T\alpha_{21}}{-1+T} + \alpha_{31} + \alpha_{41}\right) \beta_{24} + \left(\frac{\alpha_{31}}{1-T} + \alpha_{41}\right) \beta_{34} + \frac{T\alpha_{41}\beta_{44}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{12}}{1-T} + \alpha_{22} + \alpha_{32} + \alpha_{42}\right) \beta_{11} + \left(\frac{T\alpha_{22}}{-1+T} + \alpha_{32} + \alpha_{42}\right) \beta_{21} + \left(\frac{\alpha_{32}}{1-T} + \alpha_{42}\right) \beta_{31} + \frac{T\alpha_{42}\beta_{41}}{-1+T} = 1 \ \&\& \\ & \frac{1}{-1+\frac{1}{T}} + \left(\frac{\alpha_{12}}{1-T} + \alpha_{22} + \alpha_{32} + \alpha_{42}\right) \beta_{12} + \left(\frac{T\alpha_{22}}{-1+T} + \alpha_{32} + \alpha_{42}\right) \beta_{22} + \left(\frac{\alpha_{32}}{1-T} + \alpha_{42}\right) \beta_{32} + \frac{T\alpha_{42}\beta_{42}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{12}}{1-T} + \alpha_{22} + \alpha_{32} + \alpha_{42}\right) \beta_{13} + \left(\frac{T\alpha_{22}}{-1+T} + \alpha_{32} + \alpha_{42}\right) \beta_{23} + \left(\frac{\alpha_{32}}{1-T} + \alpha_{42}\right) \beta_{33} + \frac{T\alpha_{42}\beta_{43}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{12}}{1-T} + \alpha_{22} + \alpha_{32} + \alpha_{42}\right) \beta_{14} + \left(\frac{T\alpha_{22}}{-1+T} + \alpha_{32} + \alpha_{42}\right) \beta_{24} + \left(\frac{\alpha_{32}}{1-T} + \alpha_{42}\right) \beta_{34} + \frac{T\alpha_{42}\beta_{44}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{13}}{1-T} + \alpha_{23} + \alpha_{33} + \alpha_{43}\right) \beta_{11} + \left(\frac{T\alpha_{23}}{-1+T} + \alpha_{33} + \alpha_{43}\right) \beta_{21} + \left(\frac{\alpha_{33}}{1-T} + \alpha_{43}\right) \beta_{31} + \frac{T\alpha_{43}\beta_{41}}{-1+T} = 1 \ \&\& \\ & \left(\frac{\alpha_{13}}{1-T} + \alpha_{23} + \alpha_{33} + \alpha_{43}\right) \beta_{12} + \left(\frac{T\alpha_{23}}{-1+T} + \alpha_{33} + \alpha_{43}\right) \beta_{22} + \left(\frac{\alpha_{33}}{1-T} + \alpha_{43}\right) \beta_{32} + \frac{T\alpha_{43}\beta_{42}}{-1+T} = 1 \ \&\& \\ & \frac{1}{-1+T} + \left(\frac{\alpha_{13}}{1-T} + \alpha_{23} + \alpha_{33} + \alpha_{43}\right) \beta_{13} + \left(\frac{T\alpha_{23}}{-1+T} + \alpha_{33} + \alpha_{43}\right) \beta_{23} + \left(\frac{\alpha_{33}}{1-T} + \alpha_{43}\right) \beta_{33} + \frac{T\alpha_{43}\beta_{43}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{13}}{1-T} + \alpha_{23} + \alpha_{33} + \alpha_{43}\right) \beta_{14} + \left(\frac{T\alpha_{23}}{-1+T} + \alpha_{33} + \alpha_{43}\right) \beta_{24} + \left(\frac{\alpha_{33}}{1-T} + \alpha_{43}\right) \beta_{34} + \frac{T\alpha_{43}\beta_{44}}{-1+T} = 0 \ \&\& \\ & \left(\frac{\alpha_{14}}{1-T} + \alpha_{24} + \alpha_{34} + \alpha_{44}\right) \beta_{11} + \left(\frac{T\alpha_{24}}{-1+T} + \alpha_{34} + \alpha_{44}\right) \beta_{21} + \left(\frac{\alpha_{34}}{1-T} + \alpha_{44}\right) \beta_{31} + \frac{T\alpha_{44}\beta_{41}}{-1+T} = 1 \ \&\& \\ & \left(\frac{\alpha_{14}}{1-T} + \alpha_{24} + \alpha_{34} + \alpha_{44}\right) \beta_{12} + \left(\frac{T\alpha_{24}}{-1+T} + \alpha_{34} + \alpha_{44}\right) \beta_{22} + \left(\frac{\alpha_{34}}{1-T} + \alpha_{44}\right) \beta_{32} + \frac{T\alpha_{44}\beta_{42}}{-1+T} = 1 \ \&\& \\ & \left(\frac{\alpha_{14}}{1-T} + \alpha_{24} + \alpha_{34} + \alpha_{44}\right) \beta_{13} + \left(\frac{T\alpha_{24}}{-1+T} + \alpha_{34} + \alpha_{44}\right) \beta_{23} + \left(\frac{\alpha_{34}}{1-T} + \alpha_{44}\right) \beta_{33} + \frac{T\alpha_{44}\beta_{43}}{-1+T} = 1 \ \&\& \\ & \frac{1}{-1+\frac{1}{T}} + \left(\frac{\alpha_{14}}{1-T} + \alpha_{24} + \alpha_{34} + \alpha_{44}\right) \beta_{14} + \left(\frac{T\alpha_{24}}{-1+T} + \alpha_{34} + \alpha_{44}\right) \beta_{24} + \left(\frac{\alpha_{34}}{1-T} + \alpha_{44}\right) \beta_{34} + \frac{T\alpha_{44}\beta_{44}}{-1+T} = 0 \end{aligned}$$

```
unitarule = Thread[
  Flatten[Transpose[γ0[A]]] → Flatten[Simplify[
    Ωc[4].Inverse[Ωc[4].(γ0[A] /. α → β)] /. {T1|3 → T, T2|4 → 1 / T}
  ]
]
```

A very large output was generated. Here is a sample of it:

$$\left\{ \begin{aligned} &\alpha_{11} \rightarrow \\ &(\beta_{12} \beta_{24} \beta_{33} - T \beta_{12} \beta_{24} \beta_{33} - \beta_{12} \beta_{23} \beta_{34} + T \beta_{12} \beta_{23} \beta_{34} - T \beta_{24} \beta_{33} \beta_{42} + T \beta_{23} \beta_{34} \beta_{42} + \beta_{12} \beta_{24} \beta_{43} - \\ &T \beta_{12} \beta_{24} \beta_{43} + \langle\langle 10 \rangle\rangle + T \beta_{12} \beta_{23} \beta_{44} - T \beta_{23} \beta_{32} \beta_{44} - \beta_{12} \beta_{33} \beta_{44} + T \beta_{12} \beta_{33} \beta_{44} + \\ &T \beta_{22} \beta_{33} \beta_{44} + (-1 + T) \beta_{13} (\beta_{34} \beta_{42} + \beta_{24} (\beta_{32} + \beta_{42}) - \beta_{32} \beta_{44} - \beta_{22} (\beta_{34} + \beta_{44})) / \\ &(T (\beta_{12} \beta_{24} \beta_{33} \beta_{41} - \beta_{12} \beta_{23} \beta_{34} \beta_{41} - \beta_{11} \beta_{24} \beta_{33} \beta_{42} + \beta_{11} \beta_{23} \beta_{34} \beta_{42} - \beta_{12} \beta_{24} \beta_{31} \beta_{43} + \\ &\beta_{11} \beta_{24} \beta_{32} \beta_{43} + \beta_{12} \beta_{21} \beta_{34} \beta_{43} - \beta_{\langle\langle 2 \rangle\rangle} \langle\langle 3 \rangle\rangle + \beta_{14} (\langle\langle 1 \rangle\rangle) + \\ &\beta_{12} \beta_{23} \beta_{31} \beta_{44} - \beta_{11} \beta_{23} \beta_{32} \beta_{44} - \beta_{12} \beta_{21} \beta_{33} \beta_{44} + \beta_{11} \beta_{22} \beta_{33} \beta_{44} + \\ &\beta_{13} (\beta_{24} (-\beta_{32} \beta_{41} + \beta_{31} \beta_{42}) + \beta_{22} (\beta_{34} \beta_{41} - \beta_{31} \beta_{44}) + \beta_{21} (-\beta_{34} \beta_{42} + \beta_{32} \beta_{44})) / \\ \alpha_{21} \rightarrow &\frac{\beta_{14} (\langle\langle 1 \rangle\rangle) + \langle\langle 1 \rangle\rangle + \langle\langle 1 \rangle\rangle}{T (\langle\langle 1 \rangle\rangle)}, \langle\langle 12 \rangle\rangle, \alpha_{34} \rightarrow \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \\ \alpha_{44} \rightarrow &\left. \frac{\langle\langle 1 \rangle\rangle}{T (\langle\langle 1 \rangle\rangle)} \right\} \end{aligned}$$

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```
ans1 = Simplify[ $\frac{\omega / (\sigma_2 \sigma_4)}{\text{cert}@w /. \text{unitarule}}$ , eqns && ucond && ocond /. α → β]
```

$$\beta_{24} (\beta_{41} - \beta_{42}) + \beta_{23} (-\beta_{41} + \beta_{42}) + (\beta_{21} - \beta_{22}) (\beta_{43} - \beta_{44})$$

```
ans2 = Simplify[(σ2 σ4) alex / ω, eqns && ucond && ocond]
```

$$\alpha_{24} (\alpha_{41} - \alpha_{42}) + \alpha_{23} (-\alpha_{41} + \alpha_{42}) + (\alpha_{21} - \alpha_{22}) (\alpha_{43} - \alpha_{44})$$

```
ans3 = (α24 - α23) (α41 - α42) + (α21 - α22) (α43 - α44)
```

$$(-\alpha_{23} + \alpha_{24}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22}) (\alpha_{43} - \alpha_{44})$$

```
Simplify[{ans1 == ans2 /. β → α, ans2 == ans3}]
```

{True, True}