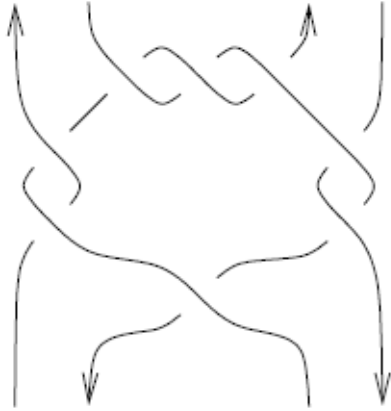


Pensieve Header: An attempt on the ribbon property using Γ-calculus; continues pensieve://2014-05/.

```
dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-06/"];
<< KnotTheory`
<< MetaCalculi/MetaCalculi-Program.m
Format[αa,b, StandardForm] := Interpretation[α10 a+b, αab];
Format[βa,b, StandardForm] := Interpretation[β10 a+b, βab];
Loading KnotTheory` version of April 3, 2014, 16:23:56.0784.
Read more at http://katlas.org/wiki/KnotTheory.
```



$$\{n = 4; \gamma_0 = \Gamma[\omega, \sum_{a=0}^n h_a \sigma_a, \sum_{a=1}^n \sum_{b=1}^n t_a h_b \alpha_{ab}], \gamma_1 = \gamma_0 // ds[2] // ds[4]\}$$

A large table representing a matrix of coefficients for the Jones polynomial. The rows are labeled with S1, S2, S3, S4, and Σ. The columns are labeled with S1, S2, S3, S4, and Σ. The entries are algebraic expressions involving α_{ij} and ω. For example, the entry for S1, S1 is $\frac{\omega(\alpha_{24}\alpha_{42} - \alpha_{22}\alpha_{44})}{\sigma_2\sigma_4}$.

```
{Ov = Xp[o1, 1] Xp[o2, 2] Xp[o3, 3] Xp[o4, 4] // Γ // dm[o1, o2, o] // dm[o, o3, o] // dm[o, o4, o],
t1 = Ov ** (γ0 * Γ[ε[o]]), t2 = (γ0 * Γ[ε[o]]) ** Ov, ocond = Simplify[t1 == t2]}
```

A table showing the result of the simplification process. It consists of three large matrices. The first matrix has rows labeled 1, s1, s2, s3, s4, so, and Σ. The second matrix has rows labeled ω, s1, s2, s3, s4, so, and Σ. The third matrix has rows labeled ω, s1, s2, s3, s4, so, and Σ. The entries are algebraic expressions involving T_o, α_{ij}, and ω.

$$\begin{aligned} (-1 + T_o) (-1 + \alpha_{11} + \alpha_{21} + \alpha_{31} + \alpha_{41}) &= 0 \ \&\& \ (-1 + T_o) (-1 + \alpha_{12} + \alpha_{22} + \alpha_{32} + \alpha_{42}) = 0 \ \&\& \\ (-1 + T_o) (-1 + \alpha_{13} + \alpha_{23} + \alpha_{33} + \alpha_{43}) &= 0 \ \&\& \ (-1 + T_o) (-1 + \alpha_{14} + \alpha_{24} + \alpha_{34} + \alpha_{44}) = 0 \end{aligned}$$

`ocond = FullSimplify[ocond /. {T0 -> 0}]`

$$\alpha_{11} + \alpha_{21} + \alpha_{31} + \alpha_{41} == 1 \ \&\& \ \alpha_{12} + \alpha_{22} + \alpha_{32} + \alpha_{42} == 1 \ \&\& \ \alpha_{13} + \alpha_{23} + \alpha_{33} + \alpha_{43} == 1 \ \&\& \ \alpha_{14} + \alpha_{24} + \alpha_{34} + \alpha_{44} == 1$$

`{U = Xm[1, u1] Xm[2, u2] Xm[3, u3] Xm[4, u4] // r // dm[u1, u2, u] // dm[u, u3, u] // dm[u, u4, u],
t1 = U** (gamma0 * r[e[u]]), t2 = (gamma0 * r[e[u]]) ** U, ucond = FullSimplify[t1 == t2]}`

$$\left(\begin{array}{cccccc} 1 & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & 1 & 0 & 0 & 0 & \frac{-1+T_1}{T_1} \\ s_2 & 0 & 1 & 0 & 0 & \frac{-1+T_2}{T_1 T_2} \\ s_3 & 0 & 0 & 1 & 0 & \frac{-1+T_3}{T_1 T_2 T_3} \\ s_4 & 0 & 0 & 0 & 1 & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & 1 & 1 & 1 & 1 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\left(\begin{array}{cccccc} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-T_2 T_3 T_4 \alpha_{11} + T_1 T_2 T_3 T_4 \alpha_{11} - T_3 T_4 \alpha_{12} + T_2 T_3 T_4 \alpha_{12} - T_4 \alpha_{13} + T_3 T_4 \alpha_{13} - \alpha_{14} + T_4 \alpha_{14}}{T_1 T_2 T_3 T_4} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-T_2 T_3 T_4 \alpha_{21} + T_1 T_2 T_3 T_4 \alpha_{21} - T_3 T_4 \alpha_{22} + T_2 T_3 T_4 \alpha_{22} - T_4 \alpha_{23} + T_3 T_4 \alpha_{23} - \alpha_{24} + T_4 \alpha_{24}}{T_1 T_2 T_3 T_4} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-T_2 T_3 T_4 \alpha_{31} + T_1 T_2 T_3 T_4 \alpha_{31} - T_3 T_4 \alpha_{32} + T_2 T_3 T_4 \alpha_{32} - T_4 \alpha_{33} + T_3 T_4 \alpha_{33} - \alpha_{34} + T_4 \alpha_{34}}{T_1 T_2 T_3 T_4} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-T_2 T_3 T_4 \alpha_{41} + T_1 T_2 T_3 T_4 \alpha_{41} - T_3 T_4 \alpha_{42} + T_2 T_3 T_4 \alpha_{42} - T_4 \alpha_{43} + T_3 T_4 \alpha_{43} - \alpha_{44} + T_4 \alpha_{44}}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\left(\begin{array}{cccccc} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-1+T_1}{T_1} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-1+T_2}{T_1 T_2} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-1+T_3}{T_1 T_2 T_3} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\frac{1}{T_1 T_2 T_3 T_4} (T_4 (T_3 ((-1 + T_1) T_2 (-1 + \alpha_{11}) + (-1 + T_2) \alpha_{12}) + (-1 + T_3) \alpha_{13}) + (-1 + T_4) \alpha_{14}) == 0 \ \&\&$$

$$\frac{1}{T_1 T_2 T_3 T_4} (T_4 (-\alpha_{23} + T_3 (1 - \alpha_{22} + T_2 (-1 + (-1 + T_1) \alpha_{21} + \alpha_{22}) + \alpha_{23})) + (-1 + T_4) \alpha_{24}) == 0 \ \&\&$$

$$\frac{1}{T_1 T_2 T_3 T_4} (-\alpha_{34} + T_4 (1 - \alpha_{33} + T_3 (-1 - \alpha_{32} + T_2 ((-1 + T_1) \alpha_{31} + \alpha_{32}) + \alpha_{33}) + \alpha_{34})) == 0 \ \&\&$$

$$\frac{1}{T_1 T_2 T_3 T_4} (1 - \alpha_{44} + T_4 (-1 - \alpha_{43} + T_3 (-\alpha_{42} + T_2 ((-1 + T_1) \alpha_{41} + \alpha_{42}) + \alpha_{43}) + \alpha_{44})) == 0 \ \&\&$$

```
{cert = γ1 // dm[1, 2, 1] // dm[3, 4, 2],
 eqns = (ε[1] ε[2] // Γ) == (cert /. σ_ → 1) // Simplify}
```

$$\left\{ \begin{array}{l} \frac{\omega (\alpha_{14} \alpha_{32} + \alpha_{24} \alpha_{32} - \alpha_{12} \alpha_{34} - \alpha_{22} \alpha_{34} + \alpha_{14} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{12} \alpha_{44} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} \\ S_1 \\ S_2 \\ \Sigma \end{array} \right. \begin{array}{l} S_1 \\ \frac{\alpha_{14} \alpha_{31} + \alpha_{24} \alpha_{31} - \alpha_{11} \alpha_{34} - \alpha_{21} \alpha_{34} + \alpha_{14} \alpha_{41} + \alpha_{24} \alpha_{41} - \alpha_{11} \alpha_{44} - \alpha_{21} \alpha_{44}}{\alpha_{14} \alpha_{32} + \alpha_{24} \alpha_{32} - \alpha_{12} \alpha_{34} - \alpha_{22} \alpha_{34} + \alpha_{14} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{12} \alpha_{44} - \alpha_{22} \alpha_{44}} \frac{\alpha_{14}}{\alpha_{14}} \\ \frac{\alpha_{12} \alpha_{31} + \alpha_{22} \alpha_{31} - \alpha_{11} \alpha_{32} - \alpha_{21} \alpha_{32} + \alpha_{12} \alpha_{41} + \alpha_{22} \alpha_{41} - \alpha_{11} \alpha_{42} - \alpha_{21} \alpha_{42}}{-\alpha_{14} \alpha_{32} - \alpha_{24} \alpha_{32} + \alpha_{12} \alpha_{34} + \alpha_{22} \alpha_{34} - \alpha_{14} \alpha_{42} - \alpha_{24} \alpha_{42} + \alpha_{12} \alpha_{44} + \alpha_{22} \alpha_{44}} \frac{\alpha_{13}}{\alpha_{14}} \\ \frac{\sigma_1}{\sigma_2} \end{array}$$

$$\left. \begin{array}{l} \frac{\alpha_{14} (\alpha_{31} + \alpha_{41}) + \alpha_{24} (\alpha_{31} + \alpha_{41}) - (\alpha_{11} + \alpha_{21}) (\alpha_{34} + \alpha_{44})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 1 \ \&\& \\ \frac{\alpha_{14} (\alpha_{33} + \alpha_{43}) + \alpha_{24} (\alpha_{33} + \alpha_{43}) - (\alpha_{13} + \alpha_{23}) (\alpha_{34} + \alpha_{44})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 0 \ \&\& \\ \frac{\alpha_{12} (\alpha_{31} + \alpha_{41}) + \alpha_{22} (\alpha_{31} + \alpha_{41}) - (\alpha_{11} + \alpha_{21}) (\alpha_{32} + \alpha_{42})}{-\alpha_{14} (\alpha_{32} + \alpha_{42}) - \alpha_{24} (\alpha_{32} + \alpha_{42}) + (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 0 \ \&\& \\ \frac{\alpha_{13} (\alpha_{32} + \alpha_{42}) + \alpha_{23} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{33} + \alpha_{43})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} == 1 \end{array} \right\}$$

```
γ1 // dm[2, 1, 1] // dm[4, 3, 3]
```

$$\left\{ \begin{array}{l} \frac{\omega (\alpha_{23} \alpha_{41} - \alpha_{24} \alpha_{41} - \alpha_{23} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{21} \alpha_{43} + \alpha_{22} \alpha_{43} + \alpha_{21} \alpha_{44} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} \\ S_1 \\ S_3 \\ \Sigma \end{array} \right. \begin{array}{l} S_1 \\ - \frac{\alpha_{13} \alpha_{41} - \alpha_{14} \alpha_{41} - \alpha_{13} \alpha_{42} + \alpha_{14} \alpha_{42} - \alpha_{11} \alpha_{43} + \alpha_{12} \alpha_{43} + \alpha_{11} \alpha_{44} - \alpha_{12} \alpha_{44}}{\alpha_{23} \alpha_{41} - \alpha_{24} \alpha_{41} - \alpha_{23} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{21} \alpha_{43} + \alpha_{22} \alpha_{43} + \alpha_{21} \alpha_{44} - \alpha_{22} \alpha_{44}} - \frac{\alpha}{\alpha} \\ - \frac{\alpha_{33} \alpha_{41} - \alpha_{34} \alpha_{41} - \alpha_{33} \alpha_{42} + \alpha_{34} \alpha_{42} - \alpha_{31} \alpha_{43} + \alpha_{32} \alpha_{43} + \alpha_{31} \alpha_{44} - \alpha_{32} \alpha_{44}}{\alpha_{23} \alpha_{41} - \alpha_{24} \alpha_{41} - \alpha_{23} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{21} \alpha_{43} + \alpha_{22} \alpha_{43} + \alpha_{21} \alpha_{44} - \alpha_{22} \alpha_{44}} - \frac{\alpha}{\alpha} \\ \frac{\sigma_1}{\sigma_2} \end{array}$$

```
alex = FullSimplify[(γ1 // dm[4, 3, 3] // dm[3, 2, 2] // dm[2, 1, 1])@ω]
```

$$\frac{1}{\sigma_2 \sigma_4} \omega (- (\alpha_{23} - \alpha_{24} + \alpha_{33} - \alpha_{34}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22} + \alpha_{31} - \alpha_{32}) (\alpha_{43} - \alpha_{44}))$$

```
{FullSimplify[alex, eqns], FullSimplify[cert@ω],
 FullSimplify[alex, eqns && ucond && ocond]}
```

$$\left\{ \frac{\omega (- (\alpha_{23} - \alpha_{24}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22}) (\alpha_{43} - \alpha_{44}))}{\sigma_2 \sigma_4}, \frac{\omega (- (\alpha_{14} + \alpha_{24}) (\alpha_{32} + \alpha_{42}) + (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44}))}{\sigma_2 \sigma_4}, \frac{\omega (- (\alpha_{23} - \alpha_{24}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22}) (\alpha_{43} - \alpha_{44}))}{\sigma_2 \sigma_4} \right\}$$

```
Ω[n_] := Ω[n] = Table[Which[i < j, 0, i == j, 1, i > j, 1 - T], {i, n}, {j, n}];
Ω[3] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 - T & 1 & 0 \\ 1 - T & 1 - T & 1 \end{pmatrix}$$

$$\begin{aligned}
& (\alpha_{23} - (-1 + T) (\alpha_{33} + \alpha_{43})) \beta_{23} + (\alpha_{33} - (-1 + T) \alpha_{43}) \beta_{33} + \alpha_{43} \beta_{43} = 1 \&\& \\
& (\alpha_{13} - (-1 + T) (\alpha_{23} + \alpha_{33} + \alpha_{43})) \beta_{14} + (\alpha_{23} - (-1 + T) (\alpha_{33} + \alpha_{43})) \beta_{24} + \\
& (\alpha_{33} - (-1 + T) \alpha_{43}) \beta_{34} + \alpha_{43} \beta_{44} = 0 \&\& T + (\alpha_{14} - (-1 + T) (\alpha_{24} + \alpha_{34} + \alpha_{44})) \beta_{11} + \\
& (\alpha_{24} - (-1 + T) (\alpha_{34} + \alpha_{44})) \beta_{21} + (\alpha_{34} - (-1 + T) \alpha_{44}) \beta_{31} + \alpha_{44} \beta_{41} = 1 \&\& \\
& T + (\alpha_{14} - (-1 + T) (\alpha_{24} + \alpha_{34} + \alpha_{44})) \beta_{12} + (\alpha_{24} - (-1 + T) (\alpha_{34} + \alpha_{44})) \beta_{22} + \\
& (\alpha_{34} - (-1 + T) \alpha_{44}) \beta_{32} + \alpha_{44} \beta_{42} = 1 \&\& T + (\alpha_{14} - (-1 + T) (\alpha_{24} + \alpha_{34} + \alpha_{44})) \beta_{13} + \\
& (\alpha_{24} - (-1 + T) (\alpha_{34} + \alpha_{44})) \beta_{23} + (\alpha_{34} - (-1 + T) \alpha_{44}) \beta_{33} + \alpha_{44} \beta_{43} = 1 \&\& \\
& (\alpha_{14} - (-1 + T) (\alpha_{24} + \alpha_{34} + \alpha_{44})) \beta_{14} + (\alpha_{24} - (-1 + T) (\alpha_{34} + \alpha_{44})) \beta_{24} + \\
& (\alpha_{34} - (-1 + T) \alpha_{44}) \beta_{34} + \alpha_{44} \beta_{44} = 1 \&\& \frac{1}{T} (T \alpha_{31} \beta_{31} - \alpha_{31} \beta_{41} + T \alpha_{31} \beta_{41} + T \alpha_{41} \beta_{41} + \\
& \alpha_{21} (T \beta_{21} + (-1 + T) (\beta_{31} + \beta_{41}))) + \alpha_{11} (T \beta_{11} + (-1 + T) (\beta_{21} + \beta_{31} + \beta_{41}))) = 1 \&\& \\
& \frac{1}{T} (T \alpha_{32} \beta_{31} - \alpha_{32} \beta_{41} + T \alpha_{32} \beta_{41} + T \alpha_{42} \beta_{41} + \alpha_{22} (T \beta_{21} + (-1 + T) (\beta_{31} + \beta_{41}))) + \\
& \alpha_{12} (T \beta_{11} + (-1 + T) (\beta_{21} + \beta_{31} + \beta_{41}))) = 0 \&\& \\
& \frac{1}{T} (T \alpha_{33} \beta_{31} - \alpha_{33} \beta_{41} + T \alpha_{33} \beta_{41} + T \alpha_{43} \beta_{41} + \alpha_{23} (T \beta_{21} + (-1 + T) (\beta_{31} + \beta_{41}))) + \\
& \alpha_{13} (T \beta_{11} + (-1 + T) (\beta_{21} + \beta_{31} + \beta_{41}))) = 0 \&\& \\
& \frac{1}{T} (T \alpha_{34} \beta_{31} - \alpha_{34} \beta_{41} + T \alpha_{34} \beta_{41} + T \alpha_{44} \beta_{41} + \alpha_{24} (T \beta_{21} + (-1 + T) (\beta_{31} + \beta_{41}))) + \\
& \alpha_{14} (T \beta_{11} + (-1 + T) (\beta_{21} + \beta_{31} + \beta_{41}))) = 0 \&\& \\
& \frac{1}{T} (1 - T + T \alpha_{31} \beta_{32} - \alpha_{31} \beta_{42} + T \alpha_{31} \beta_{42} + T \alpha_{41} \beta_{42} + \alpha_{21} (T \beta_{22} + (-1 + T) (\beta_{32} + \beta_{42}))) + \\
& \alpha_{11} (T \beta_{12} + (-1 + T) (\beta_{22} + \beta_{32} + \beta_{42}))) = 0 \&\& \\
& \frac{1}{T} (T \alpha_{32} \beta_{32} - \alpha_{32} \beta_{42} + T \alpha_{32} \beta_{42} + T \alpha_{42} \beta_{42} + \alpha_{22} (T \beta_{22} + (-1 + T) (\beta_{32} + \beta_{42}))) + \\
& \alpha_{12} (T \beta_{12} + (-1 + T) (\beta_{22} + \beta_{32} + \beta_{42}))) = 1 \&\& \\
& \frac{1}{T} (T \alpha_{33} \beta_{32} - \alpha_{33} \beta_{42} + T \alpha_{33} \beta_{42} + T \alpha_{43} \beta_{42} + \alpha_{23} (T \beta_{22} + (-1 + T) (\beta_{32} + \beta_{42}))) + \\
& \alpha_{13} (T \beta_{12} + (-1 + T) (\beta_{22} + \beta_{32} + \beta_{42}))) = 0 \&\& \\
& \frac{1}{T} (T \alpha_{34} \beta_{32} - \alpha_{34} \beta_{42} + T \alpha_{34} \beta_{42} + T \alpha_{44} \beta_{42} + \alpha_{24} (T \beta_{22} + (-1 + T) (\beta_{32} + \beta_{42}))) + \\
& \alpha_{14} (T \beta_{12} + (-1 + T) (\beta_{22} + \beta_{32} + \beta_{42}))) = 0 \&\& \\
& \frac{1}{T} (1 - T + T \alpha_{31} \beta_{33} - \alpha_{31} \beta_{43} + T \alpha_{31} \beta_{43} + T \alpha_{41} \beta_{43} + \alpha_{21} (T \beta_{23} + (-1 + T) (\beta_{33} + \beta_{43}))) + \\
& \alpha_{11} (T \beta_{13} + (-1 + T) (\beta_{23} + \beta_{33} + \beta_{43}))) = 0 \&\& \\
& \frac{1}{T} (1 - T + T \alpha_{32} \beta_{33} - \alpha_{32} \beta_{43} + T \alpha_{32} \beta_{43} + T \alpha_{42} \beta_{43} + \alpha_{22} (T \beta_{23} + (-1 + T) (\beta_{33} + \beta_{43}))) + \\
& \alpha_{12} (T \beta_{13} + (-1 + T) (\beta_{23} + \beta_{33} + \beta_{43}))) = 0 \&\& \\
& \frac{1}{T} (T \alpha_{33} \beta_{33} - \alpha_{33} \beta_{43} + T \alpha_{33} \beta_{43} + T \alpha_{43} \beta_{43} + \alpha_{23} (T \beta_{23} + (-1 + T) (\beta_{33} + \beta_{43}))) + \\
& \alpha_{13} (T \beta_{13} + (-1 + T) (\beta_{23} + \beta_{33} + \beta_{43}))) = 1 \&\& \\
& \frac{1}{T} (T \alpha_{34} \beta_{33} - \alpha_{34} \beta_{43} + T \alpha_{34} \beta_{43} + T \alpha_{44} \beta_{43} + \alpha_{24} (T \beta_{23} + (-1 + T) (\beta_{33} + \beta_{43}))) + \\
& \alpha_{14} (T \beta_{13} + (-1 + T) (\beta_{23} + \beta_{33} + \beta_{43}))) = 0 \&\& \\
& \frac{1}{T} (1 - T + T \alpha_{31} \beta_{34} - \alpha_{31} \beta_{44} + T \alpha_{31} \beta_{44} + T \alpha_{41} \beta_{44} + \alpha_{21} (T \beta_{24} + (-1 + T) (\beta_{34} + \beta_{44}))) + \\
& \alpha_{11} (T \beta_{14} + (-1 + T) (\beta_{24} + \beta_{34} + \beta_{44}))) = 0 \&\&
\end{aligned}$$

$$\frac{1}{T} (1 - T + T \alpha_{32} \beta_{34} - \alpha_{32} \beta_{44} + T \alpha_{32} \beta_{44} + T \alpha_{42} \beta_{44} + \alpha_{22} (T \beta_{24} + (-1 + T) (\beta_{34} + \beta_{44})) + \alpha_{12} (T \beta_{14} + (-1 + T) (\beta_{24} + \beta_{34} + \beta_{44}))) = 0 \&\&$$

$$\frac{1}{T} (1 - T + T \alpha_{33} \beta_{34} - \alpha_{33} \beta_{44} + T \alpha_{33} \beta_{44} + T \alpha_{43} \beta_{44} + \alpha_{23} (T \beta_{24} + (-1 + T) (\beta_{34} + \beta_{44})) + \alpha_{13} (T \beta_{14} + (-1 + T) (\beta_{24} + \beta_{34} + \beta_{44}))) = 0 \&\&$$

$$\frac{1}{T} (T \alpha_{34} \beta_{34} - \alpha_{34} \beta_{44} + T \alpha_{34} \beta_{44} + T \alpha_{44} \beta_{44} + \alpha_{24} (T \beta_{24} + (-1 + T) (\beta_{34} + \beta_{44})) + \alpha_{14} (T \beta_{14} + (-1 + T) (\beta_{24} + \beta_{34} + \beta_{44}))) = 1$$