

Pensieve Header: Finding the inner automorphisms of  $\Gamma$ , assuming constant coefficients.

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dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-06/"];
<< MetaCalculi/MetaCalculi-Program.m
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 $\alpha_{n,a,a} := 1 - \text{Sum}[\alpha_{n,b,a}, \{b, \text{Delete}[\text{Range}[n], a]\}];$ 
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$$\gamma[n_] := \Gamma\left[\omega, \sum_{a=0}^n h_a \sigma_a, \sum_{a=1}^n \sum_{b=1}^n t_a h_b \alpha_{nab}\right]$$

$\gamma[2]$

$$\begin{pmatrix} \omega & S_1 & S_2 \\ S_1 & 1 - \alpha_{2,2,1} & \alpha_{2,1,2} \\ S_2 & \alpha_{2,2,1} & 1 - \alpha_{2,1,2} \\ \Sigma & \sigma_1 & \sigma_2 \end{pmatrix}$$

$\gamma[2]$  // ds[2] // dm[1, 2, 1]

$$\begin{pmatrix} \frac{\omega}{\sigma_2} & S_1 \\ S_1 & 1 \\ \Sigma & \frac{\sigma_1}{\sigma_2} \end{pmatrix}$$

$\gamma[4]$  // ds[2] // dm[1, 2, 1]

$$\begin{pmatrix} -\frac{\omega(-1+\alpha_{4,3,2}+\alpha_{4,4,2})}{\sigma_2} & S_1 & S_3 \\ S_1 & \frac{-1+\alpha_{4,3,1}+\alpha_{4,4,1}}{-1+\alpha_{4,3,2}+\alpha_{4,4,2}} & -\frac{\alpha_{4,1,3}+\alpha_{4,2,3}}{-1+\alpha_{4,3,2}+\alpha_{4,4,2}} \\ S_3 & -\frac{\alpha_{4,3,1}-\alpha_{4,3,2}+\alpha_{4,3,2}\alpha_{4,4,1}-\alpha_{4,3,1}\alpha_{4,4,2}}{-1+\alpha_{4,3,2}+\alpha_{4,4,2}} & -\frac{1-\alpha_{4,1,3}-\alpha_{4,2,3}-\alpha_{4,3,2}-\alpha_{4,4,2}+\alpha_{4,1,3}\alpha_{4,4,2}+\alpha_{4,2,3}\alpha_{4,4,2}-\alpha_{4,4,3}+\alpha_{4,3,2}\alpha_{4,4,2}}{-1+\alpha_{4,3,2}+\alpha_{4,4,2}} \\ S_4 & \frac{-\alpha_{4,4,1}+\alpha_{4,3,2}\alpha_{4,4,1}+\alpha_{4,4,2}-\alpha_{4,3,1}\alpha_{4,4,2}}{-1+\alpha_{4,3,2}+\alpha_{4,4,2}} & \frac{\alpha_{4,1,3}\alpha_{4,4,2}+\alpha_{4,2,3}\alpha_{4,4,2}-\alpha_{4,4,3}+\alpha_{4,3,2}\alpha_{4,4,2}\alpha_{4,4,4}}{-1+\alpha_{4,3,2}+\alpha_{4,4,2}} \\ \Sigma & \frac{\sigma_1}{\sigma_2} & \sigma_3 \end{pmatrix}$$

$\alpha_{4,1,3} = -\alpha_{4,2,3}; \alpha_{4,1,4} = -\alpha_{4,2,4}; \alpha_{4,3,1} = -\alpha_{4,4,1} + \alpha_{4,3,2} + \alpha_{4,4,2}; \alpha_{4,4,1} = \alpha_{4,4,2};$

{ $\gamma[4]$ ,  $\gamma[4]$  // ds[2] // dm[1, 2, 1]}

$$\left\{ \begin{pmatrix} \omega & S_1 & S_2 & S_3 & S_4 \\ S_1 & 1 - \alpha_{4,2,1} - \alpha_{4,3,2} - \alpha_{4,4,2} & \alpha_{4,1,2} & -\alpha_{4,2,3} & -\alpha_{4,2,4} \\ S_2 & \alpha_{4,2,1} & 1 - \alpha_{4,1,2} - \alpha_{4,3,2} - \alpha_{4,4,2} & \alpha_{4,2,3} & \alpha_{4,2,4} \\ S_3 & \alpha_{4,3,2} & \alpha_{4,3,2} & 1 - \alpha_{4,4,3} & \alpha_{4,3,4} \\ S_4 & \alpha_{4,4,2} & \alpha_{4,4,2} & \alpha_{4,4,3} & 1 - \alpha_{4,3,4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} -\frac{\omega(-1+\alpha_{4,3,2}+\alpha_{4,4,2})}{\sigma_2} & S_1 & S_3 & S_4 \\ S_1 & 1 & 0 & 0 \\ S_3 & 0 & 1 - \alpha_{4,4,3} & \alpha_{4,3,4} \\ S_4 & 0 & \alpha_{4,4,3} & 1 - \alpha_{4,3,4} \\ \Sigma & \frac{\sigma_1}{\sigma_2} & \sigma_3 & \sigma_4 \end{pmatrix} \right\}$$

$\gamma[4]$  //  $ds[1]$  //  $dm[2, 1, 1]$

$$\left( \begin{array}{c} -\frac{\omega(-1+\alpha_{4,3,2}+\alpha_{4,4,2})}{\sigma_1} \\ s_1 \\ s_3 \\ s_4 \\ \Sigma \end{array} \begin{array}{cc} s_1 & s_3 \\ 1 & 0 \\ 0 & 1-\alpha_{4,4,3} \\ 0 & \alpha_{4,4,3} \end{array} \begin{array}{cc} s_4 & \\ 0 & \alpha_{4,3,4} \\ 1-\alpha_{4,3,4} & \\ \sigma_3 & \sigma_4 \end{array} \right)$$

$\{\gamma[4], \gamma[4]\}$  //  $ds[3]$  //  $dm[2, 3, 2]$

$$\left\{ \left( \begin{array}{c} \omega \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ \Sigma \end{array} \begin{array}{cc} s_1 & s_2 \\ 1-\alpha_{4,2,1}-\alpha_{4,3,2}-\alpha_{4,4,2} & \alpha_{4,1,2} \\ \alpha_{4,2,1} & 1-\alpha_{4,1,2}-\alpha_{4,3,2}-\alpha_{4,4,2} \\ \alpha_{4,3,2} & \alpha_{4,3,2} \\ \alpha_{4,4,2} & \alpha_{4,4,2} \\ \sigma_1 & \sigma_2 \end{array} \begin{array}{cc} s_3 & s_4 \\ -\alpha_{4,2,3} & -\alpha_{4,2,4} \\ \alpha_{4,2,3} & \alpha_{4,2,4} \\ 1-\alpha_{4,4,3} & \alpha_{4,3,4} \\ \alpha_{4,4,3} & 1-\alpha_{4,3,4} \\ \sigma_3 & \sigma_4 \end{array} \right), \left( \begin{array}{c} \frac{\omega(1+\alpha_{4,2,3}-\alpha_{4,4,3})}{\sigma_3} \\ s_1 \\ s_2 \\ s_4 \\ \Sigma \end{array} \right) \right.$$

$\alpha_{4,2,1} = -\alpha_{4,3,2}; \alpha_{4,2,4} = -\alpha_{4,3,4}; \alpha_{4,4,3} = \alpha_{4,1,2} + \alpha_{4,2,3} + \alpha_{4,4,2}; \alpha_{4,1,2} = -\alpha_{4,2,3};$

$\{\gamma[4], \gamma[4]\}$  //  $ds[3]$  //  $dm[2, 3, 2]$

$$\left\{ \left( \begin{array}{c} \omega \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ \Sigma \end{array} \begin{array}{cc} s_1 & s_2 \\ 1-\alpha_{4,4,2} & -\alpha_{4,2,3} \\ -\alpha_{4,3,2} & 1+\alpha_{4,2,3}-\alpha_{4,3,2}-\alpha_{4,4,2} \\ \alpha_{4,3,2} & \alpha_{4,3,2} \\ \alpha_{4,4,2} & \alpha_{4,4,2} \\ \sigma_1 & \sigma_2 \end{array} \begin{array}{cc} s_3 & s_4 \\ -\alpha_{4,2,3} & \alpha_{4,3,4} \\ \alpha_{4,2,3} & -\alpha_{4,3,4} \\ 1-\alpha_{4,4,2} & \alpha_{4,3,4} \\ \alpha_{4,4,2} & 1-\alpha_{4,3,4} \\ \sigma_3 & \sigma_4 \end{array} \right),$$

$$\left. \left( \begin{array}{c} \frac{\omega(1+\alpha_{4,2,3}-\alpha_{4,4,2})}{\sigma_3} \\ s_1 \\ s_2 \\ s_4 \\ \Sigma \end{array} \begin{array}{cc} s_1 & s_4 \\ 1-\alpha_{4,4,2} & 0 \\ 0 & 1 \\ \alpha_{4,4,2} & 0 \\ \sigma_1 & \frac{\sigma_2}{\sigma_3} \end{array} \begin{array}{cc} s_2 & s_4 \\ 0 & \alpha_{4,3,4} \\ 1 & 0 \\ 1-\alpha_{4,3,4} & \\ \sigma_3 & \sigma_4 \end{array} \right) \right\}$$

$\{\gamma[4], \gamma[4]\}$  //  $ds[4]$  //  $dm[3, 4, 3]$

$$\left\{ \left( \begin{array}{c} \omega \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ \Sigma \end{array} \begin{array}{cc} s_1 & s_2 \\ 1-\alpha_{4,4,2} & -\alpha_{4,2,3} \\ -\alpha_{4,3,2} & 1+\alpha_{4,2,3}-\alpha_{4,3,2}-\alpha_{4,4,2} \\ \alpha_{4,3,2} & \alpha_{4,3,2} \\ \alpha_{4,4,2} & \alpha_{4,4,2} \\ \sigma_1 & \sigma_2 \end{array} \begin{array}{cc} s_3 & s_4 \\ -\alpha_{4,2,3} & \alpha_{4,3,4} \\ \alpha_{4,2,3} & -\alpha_{4,3,4} \\ 1-\alpha_{4,4,2} & \alpha_{4,3,4} \\ \alpha_{4,4,2} & 1-\alpha_{4,3,4} \\ \sigma_3 & \sigma_4 \end{array} \right), \left( \begin{array}{c} \frac{\omega}{\sigma_4} \\ s_1 \\ s_2 \\ s_3 \\ \Sigma \end{array} \begin{array}{cc} s_1 & \\ 1-\alpha_{4,3,2} & \alpha_{4,3,4}-\alpha_{4,4,2}-\alpha_{4,3,4} \\ -\alpha_{4,3,2} & \alpha_{4,3,2}+\alpha_{4,3,4}+\alpha_{4,3,4} \\ \alpha_{4,3,2} & \alpha_{4,3,2}+\alpha_{4,4,2} \\ \sigma_1 & \end{array} \right) \right.$$

$$\alpha_{4,2,3} = -\alpha_{4,3,4}; \quad \alpha_{4,3,2} = -\alpha_{4,4,2};$$

$$\{\gamma[4], \gamma[4] // ds[4] // dm[3, 4, 3]\}$$

$$\left\{ \begin{array}{c} \left( \begin{array}{ccccc} \omega & s_1 & s_2 & s_3 & s_4 \\ s_1 & 1 - \alpha_{4,4,2} & \alpha_{4,3,4} & \alpha_{4,3,4} & \alpha_{4,3,4} \\ s_2 & \alpha_{4,4,2} & 1 - \alpha_{4,3,4} & -\alpha_{4,3,4} & -\alpha_{4,3,4} \\ s_3 & -\alpha_{4,4,2} & -\alpha_{4,4,2} & 1 - \alpha_{4,4,2} & \alpha_{4,3,4} \\ s_4 & \alpha_{4,4,2} & \alpha_{4,4,2} & \alpha_{4,4,2} & 1 - \alpha_{4,3,4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{array} \right), \left( \begin{array}{cccc} \frac{\omega}{\sigma_4} & s_1 & s_2 & s_3 \\ s_1 & 1 - \alpha_{4,4,2} & \alpha_{4,3,4} & 0 \\ s_2 & \alpha_{4,4,2} & 1 - \alpha_{4,3,4} & 0 \\ s_3 & 0 & 0 & 1 \\ \Sigma & \sigma_1 & \sigma_2 & \frac{\sigma_3}{\sigma_4} \end{array} \right) \end{array} \right\}$$

$$\{\gamma[4], \gamma[4] // ds[1] // dm[4, 1, 4]\}$$

$$\left\{ \begin{array}{c} \left( \begin{array}{ccccc} \omega & s_1 & s_2 & s_3 & s_4 \\ s_1 & 1 - \alpha_{4,4,2} & \alpha_{4,3,4} & \alpha_{4,3,4} & \alpha_{4,3,4} \\ s_2 & \alpha_{4,4,2} & 1 - \alpha_{4,3,4} & -\alpha_{4,3,4} & -\alpha_{4,3,4} \\ s_3 & -\alpha_{4,4,2} & -\alpha_{4,4,2} & 1 - \alpha_{4,4,2} & \alpha_{4,3,4} \\ s_4 & \alpha_{4,4,2} & \alpha_{4,4,2} & \alpha_{4,4,2} & 1 - \alpha_{4,3,4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{array} \right), \end{array} \right\}$$

$$\left\{ \begin{array}{c} \left( \begin{array}{cccc} \frac{\omega}{\sigma_1} & s_2 & s_3 & s_4 \\ s_2 & -(1 + \alpha_{4,4,2}) & (-1 + \alpha_{4,3,4} + \alpha_{4,4,2}) & -\alpha_{4,3,4} - \alpha_{4,3,4} \alpha_{4,4,2} - \alpha_{4,4,2}^2 & -\alpha_{4,3,4} - \alpha_{4,4,2} \\ s_3 & \alpha_{4,4,2} & (-1 + \alpha_{4,3,4} + \alpha_{4,4,2}) & 1 - \alpha_{4,4,2} + \alpha_{4,3,4} \alpha_{4,4,2} + \alpha_{4,4,2}^2 & \alpha_{4,3,4} + \alpha_{4,4,2} \\ s_4 & & \alpha_{4,3,4} + \alpha_{4,4,2} & \alpha_{4,3,4} + \alpha_{4,4,2} & 1 \\ \Sigma & & \sigma_2 & \sigma_3 & \frac{\sigma_4}{\sigma_1} \end{array} \right) \end{array} \right\}$$

$$\alpha_{4,3,4} = -\alpha_{4,4,2};$$

$$\{\gamma[4], \gamma[4] // ds[1] // dm[4, 1, 4]\}$$

$$\left\{ \begin{array}{c} \left( \begin{array}{ccccc} \omega & s_1 & s_2 & s_3 & s_4 \\ s_1 & 1 - \alpha_{4,4,2} & -\alpha_{4,4,2} & -\alpha_{4,4,2} & -\alpha_{4,4,2} \\ s_2 & \alpha_{4,4,2} & 1 + \alpha_{4,4,2} & \alpha_{4,4,2} & \alpha_{4,4,2} \\ s_3 & -\alpha_{4,4,2} & -\alpha_{4,4,2} & 1 - \alpha_{4,4,2} & -\alpha_{4,4,2} \\ s_4 & \alpha_{4,4,2} & \alpha_{4,4,2} & \alpha_{4,4,2} & 1 + \alpha_{4,4,2} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{array} \right), \left( \begin{array}{cccc} \frac{\omega}{\sigma_1} & s_2 & s_3 & s_4 \\ s_2 & 1 + \alpha_{4,4,2} & \alpha_{4,4,2} & 0 \\ s_3 & -\alpha_{4,4,2} & 1 - \alpha_{4,4,2} & 0 \\ s_4 & 0 & 0 & 1 \\ \Sigma & \sigma_2 & \sigma_3 & \frac{\sigma_4}{\sigma_1} \end{array} \right) \end{array} \right\}$$

$$\gamma_6 = \Gamma \left[ \omega, \sum_{a=0}^6 h_a \sigma_a, \sum_{b=1}^6 t_b h_b + \sum_{a=1}^6 \sum_{b=1}^6 t_a h_b (-1)^a \alpha \right]$$

$$\left( \begin{array}{cccccc} \omega & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ s_1 & 1 - \alpha & -\alpha & -\alpha & -\alpha & -\alpha & -\alpha \\ s_2 & \alpha & 1 + \alpha & \alpha & \alpha & \alpha & \alpha \\ s_3 & -\alpha & -\alpha & 1 - \alpha & -\alpha & -\alpha & -\alpha \\ s_4 & \alpha & \alpha & \alpha & 1 + \alpha & \alpha & \alpha \\ s_5 & -\alpha & -\alpha & -\alpha & -\alpha & 1 - \alpha & -\alpha \\ s_6 & \alpha & \alpha & \alpha & \alpha & \alpha & 1 + \alpha \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \sigma_5 & \sigma_6 \end{array} \right)$$

$\{\gamma_6 // \mathbf{dS}[2] // \mathbf{dm}[1, 2, 1], \gamma_6 // \mathbf{dS}[3] // \mathbf{dm}[2, 3, 2]\}$

$$\left\{ \begin{array}{c} \left( \begin{array}{c|cccccc} \frac{\omega}{\sigma_2} & s_1 & s_3 & s_4 & s_5 & s_6 \\ \hline s_1 & 1 & 0 & 0 & 0 & 0 \\ s_3 & 0 & 1-\alpha & -\alpha & -\alpha & -\alpha \\ s_4 & 0 & \alpha & 1+\alpha & \alpha & \alpha \\ s_5 & 0 & -\alpha & -\alpha & 1-\alpha & -\alpha \\ s_6 & 0 & \alpha & \alpha & \alpha & 1+\alpha \\ \Sigma & \frac{\sigma_1}{\sigma_2} & \sigma_3 & \sigma_4 & \sigma_5 & \sigma_6 \end{array} \right) , \\ \left( \begin{array}{c|cccccc} \frac{\omega}{\sigma_3} & s_1 & s_2 & s_4 & s_5 & s_6 \\ \hline s_1 & 1-\alpha & 0 & -\alpha & -\alpha & -\alpha \\ s_2 & 0 & 1 & 0 & 0 & 0 \\ s_4 & \alpha & 0 & 1+\alpha & \alpha & \alpha \\ s_5 & -\alpha & 0 & -\alpha & 1-\alpha & -\alpha \\ s_6 & \alpha & 0 & \alpha & \alpha & 1+\alpha \\ \Sigma & \sigma_1 & \frac{\sigma_2}{\sigma_3} & \sigma_4 & \sigma_5 & \sigma_6 \end{array} \right) \end{array} \right\}$$