Harvey: Geometry of Knot Concordance

May-28-14 5:36 AM

w/ Tim Cochran

<u>Def</u> K is concordant to J if $K \# r\bar{J}$ is smoothly slice in B^4

$\mathcal{C} := \{\text{knots}\}/\text{concordance}$ is an Abelian group under $\#$; $-K = r\bar{K}$.

<u>Def</u> A norm $\|-\|$ on a group G is

1. $\|x\| \geq 0$, $\|x\| = 0 \iff x = e$.
2. $\|xy\| \leq \|x\| + \|y\|$
3. $\|x\| = \|x^{-1}\|$

$\longrightarrow$ distance $d(x,y) = \|xy^{-1}\|$

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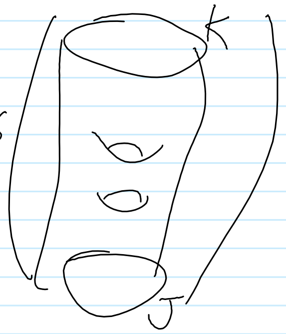
on $\mathcal{C}$:

(1) slice norm:

$\|K\|_s = \min\{g(S) : S$ is a smoothly embedded oriented surface in B^4 w/ $\partial S = K\}$

$(= g_4(K))$

$d_s(K, J) = \|K - J\|_s$ = min genus 

(2) Homology norm:

oriented simply-connected

def K is slice in a smooth 4-mfld V w/ $\partial V = S^3$ if K is $\partial\Delta$ for some smoothly embedded disk in V s.t. $[\Delta]=0$ in $H_2(V,\partial V)$

$$\|K\|_H = \min\left\{ \left(\beta_2(V) + |\sigma(V)|\right)/2 : K \text{ is slice in } V \right\}$$

β_2: 2nd Betty #, σ: signature

Def If (X, d_X) & (Y, d_Y) are metric spaces and $f: X \to Y$, we say that f is a quasi-isometry if $\exists A \geq 1, B, C \geq 0$ s.t.

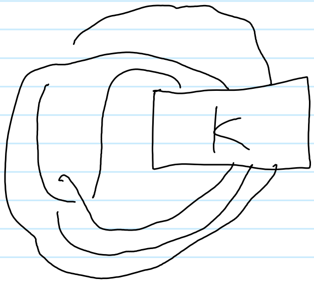
$$\frac{1}{A} d_X(x,y) - B \leq d_X(f(x), f(y)) \leq A\, d_X(x,y) + B$$

and

$$\forall z \in Y \ \exists x \in X \text{ s.t. } d_Y(z, f(x)) \leq C$$

Prop The identity map $d_S \to d_H$ on $\mathcal{C}$ is not a quasi-isometry

Pf

$P(K) :=$ 

Aru Ray: if K=trefoil

$\|P(K)\|_H = \|K\|_K$

yet

$\|P^i(K)\|_S = i+1$

Prop $\exists$ arbitrary large quasi-n-flat subspaces

<u>Prop</u> $\exists$ arbitrary large quasi-n-flat subspaces (quasi-iso to $\mathbb{R}^n$) of $\| \ \|_S$, $\| \ \|_H$

II operators on $(\mathcal{C}, d_*)$ $\quad (* = S \text{ or } H)$

Satellite operators P $\qquad w(P)$: winding # of P

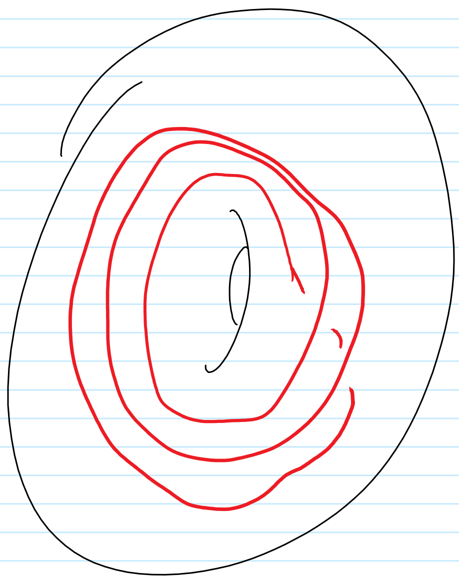
$$P : \mathcal{C} \to \mathcal{C} \quad \text{well-defined!}$$

P is usually not a group homomorphism, but it is a quasi-homomorphism:

<u>Def</u> A function $f : (\mathcal{C}, d_*) \circlearrowleft$ is a quasi-homo if $\exists \ C$ s.t.

$$\| f(K+J) - f(K) - f(J) \|_* \le C$$

<u>Pf</u> Every satellite op $\overbrace{\text{is equiv.}}^{\text{quasi-}}$ to some $(n,1)$-cable:



(meaning, any P is within a bdd distance of some $C_{n,1}$).

$\Longrightarrow$ A winding-number-0 satellite is bdd.

$\Longrightarrow$ if winding # =1, P is a quasi-isometry

<u>Thm</u> If 4-D smooth P.C. is true any P is a satellite w/ <u>strong</u> winding # ± 1,

$\Longrightarrow$ P is an isometric embedding (only for the H-norm?)

strong: The meridian of the original knot normally generates π_1 of the satellite.