Cameron Gordon: Left Orderability and Cyclic Branched Covers

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Joint w/ Tye Lidman.

A group G is "left-orderable" (LO) if $\exists$ a strict total order $<$ on G, which is left invariant:

$$g<h \Rightarrow fg<fh$$

[By convention, the trivial group is not LO]

* The reals.
* Subgroups of LO are LO.
* LO groups are torsion free.
* If G is countable then G is LO iff G is a subgroup of $\mathrm{Homeo}_+(\mathbb{R})$

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3 manifolds today are oriented & irreducible

Thm (Boyer–Rolfsen–Wiest, 2005) If M is a 3-manifold, $\pi_1(M)$ is LO iff $\pi_1(M)$ has a LO quotient.

So $H_1(M)$ infinite $\Rightarrow \pi_1(M)$ is L.O.

Q M is @HS. when is $\pi_1(M)$ L.O.?

(1) $\pi_1 M$ finite $\Rightarrow$ not LO

(2) Known for SFS.