

Pensieve Header: An attempt on the ribbon property using  $\Gamma$ -calculus.

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dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-05/"];
<< KnotTheory`
<< MetaCalculi/MetaCalculi-Program.m
Format[ $\alpha_{a,b}$ , StandardForm] := Interpretation[ $\alpha_{10^{a+b}}$ ,  $\alpha_{ab}$ ];

Loading KnotTheory` version of April 3, 2014, 16:23:56.0784.
Read more at http://katlas.org/wiki/KnotTheory.
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$$n = 4; \gamma_0 = \Gamma \left[ \omega, \sum_{a=0}^n h_a \sigma_a, \sum_{a=1}^n \sum_{b=1}^n t_a h_b \alpha_{ab} \right]$$

$$\begin{pmatrix} \omega & S_1 & S_2 & S_3 & S_4 \\ S_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ S_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ S_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ S_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{pmatrix}$$

$$\gamma_1 = (\gamma_0 // ds[2] // ds[4])$$

$$\begin{pmatrix} \frac{\omega (\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} & & & & & \\ & S_1 & & & & S_2 \\ S_1 & \frac{\alpha_{14} \alpha_{22} \alpha_{41} - \alpha_{12} \alpha_{24} \alpha_{41} - \alpha_{14} \alpha_{21} \alpha_{42} + \alpha_{11} \alpha_{24} \alpha_{42} + \alpha_{12} \alpha_{21} \alpha_{44} - \alpha_{11} \alpha_{22} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & & & - \frac{\alpha_{14} \alpha_{42} - \alpha_{12} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & - \frac{-\alpha_{14} \alpha_{23} \alpha_{42} + \alpha_{13} \alpha_{24} \alpha_{42} + \alpha_{11}}{\alpha_{44}} \\ S_2 & \frac{\alpha_{24} \alpha_{41} - \alpha_{21} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & & & \frac{\alpha_{44}}{\alpha_{44}} & \\ S_3 & \frac{-\alpha_{24} \alpha_{32} \alpha_{41} + \alpha_{22} \alpha_{34} \alpha_{41} + \alpha_{24} \alpha_{31} \alpha_{42} - \alpha_{21} \alpha_{34} \alpha_{42} - \alpha_{22} \alpha_{31} \alpha_{44} + \alpha_{21} \alpha_{32} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & & & - \frac{\alpha_{34} \alpha_{42} - \alpha_{32} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & \frac{\alpha_{24} \alpha_{33} \alpha_{42} - \alpha_{23} \alpha_{34} \alpha_{42} - \alpha_{24}}{\alpha_{44}} \\ S_4 & \frac{\alpha_{22} \alpha_{41} - \alpha_{21} \alpha_{42}}{-\alpha_{24} \alpha_{42} + \alpha_{22} \alpha_{44}} & & & \frac{\alpha_{42}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & \\ \Sigma & \sigma_1 & & & \frac{1}{\sigma_2} & \end{pmatrix}$$

$$Ov = Xp[o1, 1] Xp[o2, 2] Xp[o3, 3] Xp[o4, 4] // \Gamma // dm[o1, o2, o] // dm[o, o3, o] // dm[o, o4, o]$$

$$\begin{pmatrix} 1 & S_1 & S_2 & S_3 & S_4 & S_o \\ S_1 & T_o & 0 & 0 & 0 & 0 \\ S_2 & 0 & T_o & 0 & 0 & 0 \\ S_3 & 0 & 0 & T_o & 0 & 0 \\ S_4 & 0 & 0 & 0 & T_o & 0 \\ S_o & 1 - T_o & 1 - T_o & 1 - T_o & 1 - T_o & 1 \\ \Sigma & T_o & T_o & T_o & T_o & 1 \end{pmatrix}$$

`{t1 = Ov** (γ1 * Γ[ε[o]]), t2 = (γ1 * Γ[ε[o]]) ** Ov, ocond = Simplify[t1 == t2]}`

$$\left\{ \begin{array}{l} -\frac{\omega (\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_o \\ \Sigma \end{array} \right. \left\{ \begin{array}{l} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_o \\ \Sigma \end{array} \right.$$

$$\frac{1}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} (-1 + T_o) (\alpha_{24} (-(-1 + \alpha_{12} + \alpha_{32}) \alpha_{41} + (-1 + \alpha_{11} + \alpha_{31}) \alpha_{42}) + \alpha_{22} ((-1 + \alpha_{14} + \alpha_{34}) \alpha_{41} - (-1 + \alpha_{11} + \alpha_{31}) \alpha_{44}) + \alpha_{21} (-(-1 + \alpha_{14} + \alpha_{34}) \alpha_{42} + (-1 + \alpha_{12} + \alpha_{32}) \alpha_{44})) = 0 \&\&$$

$$T_o + \frac{(-1 + T_o) ((-1 + \alpha_{14} + \alpha_{34}) \alpha_{42} - (-1 + \alpha_{12} + \alpha_{32}) \alpha_{44})}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} = 1 \&\&$$

$$\frac{1}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} (-1 + T_o) (\alpha_{24} ((-1 + \alpha_{13} + \alpha_{33}) \alpha_{42} - (-1 + \alpha_{12} + \alpha_{32}) \alpha_{43}) + \alpha_{23} (-(-1 + \alpha_{14} + \alpha_{34}) \alpha_{42} + (-1 + \alpha_{12} + \alpha_{32}) \alpha_{44}) + \alpha_{22} ((-1 + \alpha_{14} + \alpha_{34}) \alpha_{43} - (-1 + \alpha_{13} + \alpha_{33}) \alpha_{44})) = 0 \&\&$$

$$T_o + \frac{(-1 + T_o) (-\alpha_{24} (-1 + \alpha_{12} + \alpha_{32}) + \alpha_{22} (-1 + \alpha_{14} + \alpha_{34}))}{-\alpha_{24} \alpha_{42} + \alpha_{22} \alpha_{44}} = 1 \}$$

`ocond =`

$$\text{FullSimplify}\left[\text{FullSimplify}\left[\text{ocond} /. \left\{T_o \rightarrow 0, \alpha_{a_n} \Rightarrow 1 - \sum_{b=1}^{n-1} \alpha_{ab}\right\}\right] /. \frac{x}{y} = z \Rightarrow x = y z\right]$$

$$1 + \alpha_{12} (-1 + \alpha_{41} + \alpha_{43}) + \alpha_{22} (-1 + \alpha_{41} + \alpha_{43}) + \alpha_{32} (-1 + \alpha_{41} + \alpha_{43}) =$$

$$\alpha_{41} + (-1 + \alpha_{11} + \alpha_{13} + \alpha_{21} + \alpha_{23} + \alpha_{31} + \alpha_{33}) \alpha_{42} + \alpha_{43} \&\&$$

$$\alpha_{23} ((-1 + \alpha_{12} + \alpha_{32}) (-1 + \alpha_{41}) - (1 + \alpha_{11} + \alpha_{31}) \alpha_{42}) +$$

$$\alpha_{22} (-(-1 + \alpha_{13} + \alpha_{33}) (-1 + \alpha_{41}) + (1 + \alpha_{11} + \alpha_{31}) \alpha_{43}) +$$

$$(-1 + \alpha_{21}) ((-1 + \alpha_{13} + \alpha_{33}) \alpha_{42} - (-1 + \alpha_{12} + \alpha_{32}) \alpha_{43}) = 0 \&\&$$

$$(-1 + \alpha_{21} + \alpha_{23}) (-1 + \alpha_{12} + \alpha_{32} + \alpha_{42}) = \alpha_{22} (-1 + \alpha_{11} + \alpha_{13} + \alpha_{31} + \alpha_{33} + \alpha_{41} + \alpha_{43})$$

`U = Xm[1, u1] Xm[2, u2] Xm[3, u3] Xm[4, u4] // Γ // dm[u1, u2, u] // dm[u, u3, u] // dm[u, u4, u]`

$$\left( \begin{array}{cccccc} 1 & S_1 & S_2 & S_3 & S_4 & S_u \\ S_1 & 1 & 0 & 0 & 0 & \frac{-1+T_1}{T_1} \\ S_2 & 0 & 1 & 0 & 0 & \frac{-1+T_2}{T_1 T_2} \\ S_3 & 0 & 0 & 1 & 0 & \frac{-1+T_3}{T_1 T_2 T_3} \\ S_4 & 0 & 0 & 0 & 1 & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ S_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & 1 & 1 & 1 & 1 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right)$$

`{t1 = U** (γ0 * Γ[ε[u]]), t2 = (γ0 * Γ[ε[u]]) ** U, ucond = Simplify[t1 == t2]}`

$$\left( \begin{array}{cccc|c} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-T_2 T_3 T_4 \alpha_{11} + T_1 T_2 T_3 T_4 \alpha_{11} - T_3 T_4 \alpha_{12} + T_2 T_3 T_4 \alpha_{12} - T_4 \alpha_{13} + T_3 T_4 \alpha_{13} - \alpha_{14} + T_4 \alpha_{14}}{T_1 T_2 T_3 T_4} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-T_2 T_3 T_4 \alpha_{21} + T_1 T_2 T_3 T_4 \alpha_{21} - T_3 T_4 \alpha_{22} + T_2 T_3 T_4 \alpha_{22} - T_4 \alpha_{23} + T_3 T_4 \alpha_{23} - \alpha_{24} + T_4 \alpha_{24}}{T_1 T_2 T_3 T_4} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-T_2 T_3 T_4 \alpha_{31} + T_1 T_2 T_3 T_4 \alpha_{31} - T_3 T_4 \alpha_{32} + T_2 T_3 T_4 \alpha_{32} - T_4 \alpha_{33} + T_3 T_4 \alpha_{33} - \alpha_{34} + T_4 \alpha_{34}}{T_1 T_2 T_3 T_4} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-T_2 T_3 T_4 \alpha_{41} + T_1 T_2 T_3 T_4 \alpha_{41} - T_3 T_4 \alpha_{42} + T_2 T_3 T_4 \alpha_{42} - T_4 \alpha_{43} + T_3 T_4 \alpha_{43} - \alpha_{44} + T_4 \alpha_{44}}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\left( \begin{array}{cccc|c} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-1+T_1}{T_1} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-1+T_2}{T_1 T_2} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-1+T_3}{T_1 T_2 T_3} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array} \right),$$

$$\left. \begin{aligned} & \frac{1}{T_1 T_2 T_3 T_4} (T_2 T_3 T_4 (1 + (-1 + T_1) \alpha_{11} + \alpha_{12}) - T_4 \alpha_{13} + T_3 T_4 (-\alpha_{12} + \alpha_{13}) - \alpha_{14} + T_4 \alpha_{14}) == 1 \ \&\& \\ & \frac{1}{T_1 T_2 T_3 T_4} (T_3 T_4 (1 - \alpha_{22} + T_2 (-1 + (-1 + T_1) \alpha_{21} + \alpha_{22}) + \alpha_{23}) - \alpha_{24} + T_4 (-\alpha_{23} + \alpha_{24})) == 0 \ \&\& \\ & \frac{1}{T_1 T_2 T_3 T_4} (-\alpha_{34} + T_4 (1 - \alpha_{33} + T_3 (-1 - \alpha_{32} + T_2 ((-1 + T_1) \alpha_{31} + \alpha_{32}) + \alpha_{33}) + \alpha_{34})) == 0 \ \&\& \\ & \frac{1}{T_1 T_2 T_3 T_4} (1 - \alpha_{44} + T_4 (-1 + T_2 T_3 ((-1 + T_1) \alpha_{41} + \alpha_{42}) - \alpha_{43} + T_3 (-\alpha_{42} + \alpha_{43}) + \alpha_{44})) == 0 \end{aligned} \right\}$$

`cert = γ1 // dm[1, 2, 1] // dm[3, 4, 2]`

$$\left( \begin{array}{c|c} \frac{\omega (\alpha_{14} \alpha_{32} + \alpha_{24} \alpha_{32} - \alpha_{12} \alpha_{34} - \alpha_{22} \alpha_{34} + \alpha_{14} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{12} \alpha_{44} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} & s_1 \\ s_1 & \frac{\alpha_{14} \alpha_{31} + \alpha_{24} \alpha_{31} - \alpha_{11} \alpha_{34} - \alpha_{21} \alpha_{34} + \alpha_{14} \alpha_{41} + \alpha_{24} \alpha_{41} - \alpha_{11} \alpha_{44} - \alpha_{21} \alpha_{44}}{\alpha_{14} \alpha_{32} + \alpha_{24} \alpha_{32} - \alpha_{12} \alpha_{34} - \alpha_{22} \alpha_{34} + \alpha_{14} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{12} \alpha_{44} - \alpha_{22} \alpha_{44}} \\ s_2 & \frac{\alpha_{12} \alpha_{31} + \alpha_{22} \alpha_{31} - \alpha_{11} \alpha_{32} - \alpha_{21} \alpha_{32} + \alpha_{12} \alpha_{41} + \alpha_{22} \alpha_{41} - \alpha_{11} \alpha_{42} - \alpha_{21} \alpha_{42}}{-\alpha_{14} \alpha_{32} - \alpha_{24} \alpha_{32} + \alpha_{12} \alpha_{34} + \alpha_{22} \alpha_{34} - \alpha_{14} \alpha_{42} - \alpha_{24} \alpha_{42} + \alpha_{12} \alpha_{44} + \alpha_{22} \alpha_{44}} \\ \Sigma & \frac{\sigma_1}{\sigma_2} \end{array} \right)$$

**eqns = (e[1] e[2] // Γ) == (cert /. σ<sub>-</sub> → 1) // Simplify**

$$\begin{aligned}
 &-\omega (\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})) = 1 \ \&\& \\
 &\frac{\alpha_{14} (\alpha_{31} + \alpha_{41}) + \alpha_{24} (\alpha_{31} + \alpha_{41}) - (\alpha_{11} + \alpha_{21}) (\alpha_{34} + \alpha_{44})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} = 1 \ \&\& \\
 &\frac{\alpha_{14} (\alpha_{33} + \alpha_{43}) + \alpha_{24} (\alpha_{33} + \alpha_{43}) - (\alpha_{13} + \alpha_{23}) (\alpha_{34} + \alpha_{44})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} = 0 \ \&\& \\
 &\frac{\alpha_{12} (\alpha_{31} + \alpha_{41}) + \alpha_{22} (\alpha_{31} + \alpha_{41}) - (\alpha_{11} + \alpha_{21}) (\alpha_{32} + \alpha_{42})}{- \alpha_{14} (\alpha_{32} + \alpha_{42}) - \alpha_{24} (\alpha_{32} + \alpha_{42}) + (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} = 0 \ \&\& \\
 &\frac{\alpha_{13} (\alpha_{32} + \alpha_{42}) + \alpha_{23} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{33} + \alpha_{43})}{\alpha_{14} (\alpha_{32} + \alpha_{42}) + \alpha_{24} (\alpha_{32} + \alpha_{42}) - (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44})} = 1
 \end{aligned}$$

**alex = (γ1 // dm[4, 3, 3] // dm[3, 2, 2] // dm[2, 1, 1])[[1]]**

$$\begin{aligned}
 &-\frac{1}{\sigma_2 \sigma_4} \omega (\alpha_{23} \alpha_{41} - \alpha_{24} \alpha_{41} + \alpha_{33} \alpha_{41} - \alpha_{34} \alpha_{41} - \alpha_{23} \alpha_{42} + \alpha_{24} \alpha_{42} - \alpha_{33} \alpha_{42} + \\
 &\quad \alpha_{34} \alpha_{42} - \alpha_{21} \alpha_{43} + \alpha_{22} \alpha_{43} - \alpha_{31} \alpha_{43} + \alpha_{32} \alpha_{43} + \alpha_{21} \alpha_{44} - \alpha_{22} \alpha_{44} + \alpha_{31} \alpha_{44} - \alpha_{32} \alpha_{44})
 \end{aligned}$$

**FullSimplify[alex, eqns && ucond && ocond]**

$$\frac{\omega (- (\alpha_{23} - \alpha_{24}) (\alpha_{41} - \alpha_{42}) + (\alpha_{21} - \alpha_{22}) (\alpha_{43} - \alpha_{44}))}{\sigma_2 \sigma_4}$$

**FullSimplify[cert[[1]]]**

$$\frac{\omega (- (\alpha_{14} + \alpha_{24}) (\alpha_{32} + \alpha_{42}) + (\alpha_{12} + \alpha_{22}) (\alpha_{34} + \alpha_{44}))}{\sigma_2 \sigma_4}$$

$$\mathbf{M} = \begin{pmatrix} -\frac{\frac{T_2}{-T_1 - T_2 + T_1 T_2}}{\frac{T_1 (-1 + T_2) T_3}{(-T_1 - T_2 + T_1 T_2) (-T_2 - T_3 + T_2 T_3)}} & \frac{\frac{-1 + T_1}{-T_1 - T_2 + T_1 T_2}}{\frac{(1 - 2 T_1 - T_2 + T_1 T_2) T_3}{(-T_1 - T_2 + T_1 T_2) (-T_2 - T_3 + T_2 T_3)}} & 0 \\ \frac{\frac{T_1 (-1 + T_2) T_2 (-1 + T_3)}{(-T_1 - T_2 + T_1 T_2) (-T_2 - T_3 + T_2 T_3)}}{\frac{T_2 (1 - 2 T_1 - T_2 + T_1 T_2) (-1 + T_3)}{(-T_1 - T_2 + T_1 T_2) (-T_2 - T_3 + T_2 T_3)}} & \frac{\frac{1 - 2 T_2 - T_3 + T_2 T_3}{-T_2 - T_3 + T_2 T_3}}{\frac{1 - 2 T_2 - T_3 + T_2 T_3}{-T_2 - T_3 + T_2 T_3}} \end{pmatrix};$$

**Simplify[M /. T<sub>a</sub> → 1 / T<sub>a</sub>] // MatrixForm**

$$\begin{pmatrix} \frac{T_1}{-1 + T_1 + T_2} & \frac{(-1 + T_1) T_2}{-1 + T_1 + T_2} & 0 \\ \frac{(-1 + T_2) T_2}{(-1 + T_1 + T_2) (-1 + T_2 + T_3)} & -\frac{(1 + T_1) (-1 + T_2) - 2 T_2}{(-1 + T_1 + T_2) (-1 + T_2 + T_3)} & \frac{(-1 + T_2) T_3}{-1 + T_2 + T_3} \\ \frac{(-1 + T_2) (-1 + T_3)}{(-1 + T_1 + T_2) (-1 + T_2 + T_3)} & -\frac{(1 + T_1) (-1 + T_2) - 2 T_2}{(-1 + T_1 + T_2) (-1 + T_2 + T_3)} & \frac{-1 - T_2 (-1 + T_3) + 2 T_3}{-1 + T_2 + T_3} \end{pmatrix}$$

**Simplify[(M /. T<sub>a</sub> → 1 / T<sub>a</sub>).Transpose[M]] // Simplify // MatrixForm**

$$\begin{pmatrix} \frac{(1 - 3 T_1 + T_1^2) T_2}{(T_1 (-1 + T_2) - T_2) (-1 + T_1 + T_2)} & -\frac{(T_1 (3 - 2 T_2) T_2 + (T_1 (-1 + T_2) - T_2))}{(T_1 (-1 + T_2) - T_2)} \\ -\frac{T_2 (-1 + T_1) (2 - 3 T_2) + T_1^2 (-1 + T_2) + T_2 + T_2^2}{(T_1 (-1 + T_2) - T_2) (-1 + T_1 + T_2) (-1 + T_2 + T_3)} & \frac{(-T_2 (-2 + 6 T_2 - 5 T_2^2 + T_2^3) + T_1^2 (-1 + 5 (T_1 (-1 + T_2) - T_2) (-1 + T_2 + T_3)))}{(T_1 (-1 + T_2) - T_2) (-1 + T_2 + T_3)} \\ -\frac{(-1 + T_1) (2 - 3 T_2) + T_1^2 (-1 + T_2) + T_2 + T_2^2}{(T_1 (-1 + T_2) - T_2) (-1 + T_1 + T_2) (-1 + T_2 + T_3)} & \frac{-T_1^2 (-1 + T_2) (-1 + T_2^2 (-1 + T_3) + 2 T_2^2 - T_2 (-2 + 2 T_3 + T_3^2)) + (-1 + T_2) (T_2^2 (2 - 3 T_3) + T_2^3 (-1 + T_3) - (T_1 (-1 + T_2) - T_2) (-1 + T_2 + T_3))}{(T_1 (-1 + T_2) - T_2) (-1 + T_2 + T_3)} \end{pmatrix}$$

`Simplify[M /. T_a_ -> 1 / T_a] / M /. T_ -> T // Simplify // MatrixForm`

Power::infy : Infinite expression  $\frac{1}{0}$  encountered. >>

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. >>

$$\begin{pmatrix} -\frac{(-2+T) T}{-1+2 T} & \frac{(-2+T) T^2}{-1+2 T} & \text{Indeterminate} \\ -\frac{(-2+T)^2 T}{(1-2 T)^2} & \frac{(-2+T)^2 T^2}{(1-2 T)^2} & \frac{(-2+T) T^2}{-1+2 T} \\ \frac{(-2+T)^2}{(1-2 T)^2} & -\frac{(-2+T)^2 T}{(1-2 T)^2} & -\frac{(-2+T) T}{-1+2 T} \end{pmatrix}$$