

Pensieve Header: An attempt on the ribbon property using Γ -calculus.

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dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-05/"];
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<< KnotTheory`
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<< MetaCalculi/MetaCalculi-Program.m
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Loading KnotTheory` version of April 3, 2014, 16:23:56.0784.

Read more at <http://katlas.org/wiki/KnotTheory>.

$$n = 4; \gamma_0 = \Gamma \left[\omega, \sum_{a=0}^n h_a \sigma_a, \sum_{a=1}^n \sum_{b=1}^n t_a h_b \alpha_{10\ a+b}[T_1, T_2, T_3, T_4] \right]$$

$$\begin{pmatrix} \omega & s_1 & s_2 & s_3 & s_4 \\ s_1 & \alpha_{11}[T_1, T_2, T_3, T_4] & \alpha_{12}[T_1, T_2, T_3, T_4] & \alpha_{13}[T_1, T_2, T_3, T_4] & \alpha_{14}[T_1, T_2, T_3, T_4] \\ s_2 & \alpha_{21}[T_1, T_2, T_3, T_4] & \alpha_{22}[T_1, T_2, T_3, T_4] & \alpha_{23}[T_1, T_2, T_3, T_4] & \alpha_{24}[T_1, T_2, T_3, T_4] \\ s_3 & \alpha_{31}[T_1, T_2, T_3, T_4] & \alpha_{32}[T_1, T_2, T_3, T_4] & \alpha_{33}[T_1, T_2, T_3, T_4] & \alpha_{34}[T_1, T_2, T_3, T_4] \\ s_4 & \alpha_{41}[T_1, T_2, T_3, T_4] & \alpha_{42}[T_1, T_2, T_3, T_4] & \alpha_{43}[T_1, T_2, T_3, T_4] & \alpha_{44}[T_1, T_2, T_3, T_4] \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{pmatrix}$$

```
U = Xm[1, u1] Xm[2, u2] Xm[3, u3] Xm[4, u4] // \Gamma // dm[u1, u2, u] // dm[u, u3, u] // dm[u, u4, u]
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$$\begin{pmatrix} 1 & s_1 & s_2 & s_3 & s_4 & s_u \\ s_1 & 1 & 0 & 0 & 0 & \frac{-1+T_1}{T_1} \\ s_2 & 0 & 1 & 0 & 0 & \frac{-1+T_2}{T_1 T_2} \\ s_3 & 0 & 0 & 1 & 0 & \frac{-1+T_3}{T_1 T_2 T_3} \\ s_4 & 0 & 0 & 0 & 1 & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & 1 & 1 & 1 & 1 & \frac{1}{T_1 T_2 T_3 T_4} \end{pmatrix}$$

`{t1 = U** (γ0 * Γ[ε[u]]), t2 = (γ0 * Γ[ε[u]]) ** U, ucond = Simplify[t1 == t2]}`

$$\left(\begin{array}{ccccc} \omega & s_1 & s_2 & s_3 & s_4 \\ s_1 & \alpha_{11}[T_1, T_2, T_3, T_4] & \alpha_{12}[T_1, T_2, T_3, T_4] & \alpha_{13}[T_1, T_2, T_3, T_4] & \alpha_{14}[T_1, T_2, T_3, T_4] & \frac{-T_2 T_3 T_4 \alpha_{11}[T_1, T_2, T_3, T_4]}{T_1 T_2 T_3 T_4} \\ s_2 & \alpha_{21}[T_1, T_2, T_3, T_4] & \alpha_{22}[T_1, T_2, T_3, T_4] & \alpha_{23}[T_1, T_2, T_3, T_4] & \alpha_{24}[T_1, T_2, T_3, T_4] & \frac{-T_2 T_3 T_4 \alpha_{21}[T_1, T_2, T_3, T_4]}{T_1 T_2 T_3 T_4} \\ s_3 & \alpha_{31}[T_1, T_2, T_3, T_4] & \alpha_{32}[T_1, T_2, T_3, T_4] & \alpha_{33}[T_1, T_2, T_3, T_4] & \alpha_{34}[T_1, T_2, T_3, T_4] & \frac{-T_2 T_3 T_4 \alpha_{31}[T_1, T_2, T_3, T_4]}{T_1 T_2 T_3 T_4} \\ s_4 & \alpha_{41}[T_1, T_2, T_3, T_4] & \alpha_{42}[T_1, T_2, T_3, T_4] & \alpha_{43}[T_1, T_2, T_3, T_4] & \alpha_{44}[T_1, T_2, T_3, T_4] & \frac{-T_2 T_3 T_4 \alpha_{41}[T_1, T_2, T_3, T_4]}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \end{array} \right)$$

$$\begin{aligned} & \frac{1}{T_1 T_2 T_3 T_4} (T_2 T_3 T_4 (1 + (-1 + T_1) \alpha_{11}[T_1, T_2, T_3, T_4] + \alpha_{12}[T_1, T_2, T_3, T_4]) - T_4 \alpha_{13}[T_1, T_2, T_3, T_4] \\ & \quad + T_4 \alpha_{14}[T_1, T_2, T_3, T_4]) + T_3 T_4 (-\alpha_{12}[T_1, T_2, T_3, T_4] + \alpha_{13}[T_1, T_2, T_3, T_4]) - \alpha_{14}[T_1, T_2, T_3, T_4] + \\ & \quad T_4 \alpha_{14}[T_1, T_2, T_3, T_4]) = 1 \ \&\& \frac{1}{T_1 T_2 T_3 T_4} (T_3 T_4 (1 - \alpha_{22}[T_1, T_2, T_3, T_4] + \\ & \quad T_2 (-1 + (-1 + T_1) \alpha_{21}[T_1, T_2, T_3, T_4] + \alpha_{22}[T_1, T_2, T_3, T_4]) + \alpha_{23}[T_1, T_2, T_3, T_4]) - \\ & \quad \alpha_{24}[T_1, T_2, T_3, T_4] + T_4 (-\alpha_{23}[T_1, T_2, T_3, T_4] + \alpha_{24}[T_1, T_2, T_3, T_4])) = 0 \ \&\& \\ & \frac{1}{T_1 T_2 T_3 T_4} (-\alpha_{34}[T_1, T_2, T_3, T_4] + T_4 (1 - \alpha_{33}[T_1, T_2, T_3, T_4] + T_3 (-1 - \alpha_{32}[T_1, T_2, T_3, T_4] + \\ & \quad T_2 ((-1 + T_1) \alpha_{31}[T_1, T_2, T_3, T_4] + \alpha_{32}[T_1, T_2, T_3, T_4]) + \alpha_{33}[T_1, T_2, T_3, T_4]) + \\ & \quad \alpha_{34}[T_1, T_2, T_3, T_4])) = 0 \ \&\& \frac{1}{T_1 T_2 T_3 T_4} (1 - \alpha_{44}[T_1, T_2, T_3, T_4] + \\ & \quad T_4 (-1 + T_2 T_3 ((-1 + T_1) \alpha_{41}[T_1, T_2, T_3, T_4] + \alpha_{42}[T_1, T_2, T_3, T_4]) - \alpha_{43}[T_1, T_2, T_3, T_4] + \\ & \quad T_3 (-\alpha_{42}[T_1, T_2, T_3, T_4] + \alpha_{43}[T_1, T_2, T_3, T_4]) + \alpha_{44}[T_1, T_2, T_3, T_4])) = 0 \end{aligned}$$

`γ1 = (γ0 // ds[2] // ds[4]) /. T_ → T`

$$\left(\begin{array}{ccccc} \omega & \frac{\alpha_{24}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{42}[T, \frac{1}{T}, T, \frac{1}{T}] - \alpha_{22}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{44}[T, \frac{1}{T}, T, \frac{1}{T}]}{\sigma_2 \sigma_4} & & & \\ s_1 & & \frac{\alpha_{14}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{22}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{41}[T, \frac{1}{T}, T, \frac{1}{T}] - \alpha_{12}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{24}[T, \frac{1}{T}, T, \frac{1}{T}]}{\sigma_2 \sigma_4} & & \\ s_2 & & & & \\ s_3 & & \frac{-\alpha_{24}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{32}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{41}[T, \frac{1}{T}, T, \frac{1}{T}] + \alpha_{22}[T, \frac{1}{T}, T, \frac{1}{T}] \alpha_{34}[T, \frac{1}{T}, T, \frac{1}{T}]}{\sigma_2 \sigma_4} & & \\ s_4 & & & & \\ \Sigma & & & & \end{array} \right)$$

cert = γ_1 // dm[1, 2, 1] // dm[3, 4, 2]

$$\left(\frac{\omega \left(\alpha_{14} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{24} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] - \alpha_{12} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \alpha_{34} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] - \alpha_{22} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \alpha_{34} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{14} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right)}{\sigma_2 \sigma_4} \right)$$

S₁

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Σ

eqns = (ε[1] ε[2] // Γ) == (cert /. σ_ → 1) // Simplify

$$\begin{aligned} & -\omega \left(\alpha_{14} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{42} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) + \right. \\ & \quad \alpha_{24} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{42} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) - \\ & \quad \left. \left(\alpha_{12} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{22} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \left(\alpha_{34} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{44} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \right) = 1 \&\& \\ & \left(\alpha_{14} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{31} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{41} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) + \right. \\ & \quad \alpha_{24} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{31} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{41} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) - \\ & \quad \left. \left(\alpha_{11} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{21} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \left(\alpha_{34} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{44} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \right) / \\ & \left(\alpha_{14} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{42} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) + \right. \\ & \quad \alpha_{24} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{42} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) - \\ & \quad \left. \left(\alpha_{12} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{22} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \left(\alpha_{34} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{44} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \right) = 1 \&\& \\ & \left(\alpha_{14} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{33} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{43} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) + \right. \\ & \quad \alpha_{24} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{33} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{43} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) - \\ & \quad \left. \left(\alpha_{13} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{23} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \left(\alpha_{34} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{44} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \right) / \\ & \left(\alpha_{14} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{42} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) + \right. \\ & \quad \alpha_{24} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{32} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{42} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) - \\ & \quad \left. \left(\alpha_{12} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{22} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \left(\alpha_{34} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{44} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \right) = 0 \&\& \\ & \left(\alpha_{12} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{31} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{41} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) + \right. \\ & \quad \left. \alpha_{22} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \left(\alpha_{31} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] + \alpha_{41} \left[T, \frac{1}{T}, T, \frac{1}{T} \right] \right) \right) - \end{aligned}$$

FullSimplify[cert[[1]]]

$$\frac{1}{\sigma_2 \sigma_4} \omega \left(- \left(\alpha_{14} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] + \alpha_{24} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] \right) \left(\alpha_{32} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] + \alpha_{42} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] \right) + \right. \\ \left. \left(\alpha_{12} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] + \alpha_{22} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] \right) \left(\alpha_{34} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] + \alpha_{44} \left[\mathbb{T}, \frac{1}{\mathbb{T}}, \mathbb{T}, \frac{1}{\mathbb{T}} \right] \right) \right)$$