Jordan: How to integrate the quantum group over a surface

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How to make sense of $\int_S U_q\mathfrak{g}\text{-mod}$?

Let H be a quasitriangular Hopf algebra:

$\exists\ \sigma_{V,W}: V\otimes W \xrightarrow{\sim} W\otimes V$

$\mathcal{A}$: category of H-modules - a braided tensor category, an "E_2-algebra".

Example: $H = U_q\mathfrak{g}$ $\mathcal{A}=\mathcal{A}_q$

Braid groups: $V \in \mathrm{Ob}\,\mathcal{A}$

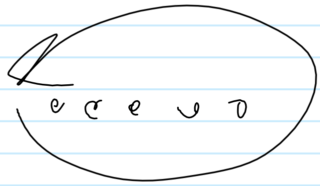
$V^{\otimes n} \circlearrowleft B_n = B_n(\mathbb{R}^2) = \langle T_1, \dots, T_{n-1}\rangle$

Braid groups of other surfaces: S: a real surface, may be punctured.

$\mathrm{Conf}_n(S) = \{X \subset S : |X| = n\}$

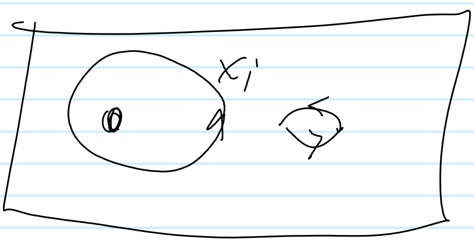
$B_n(S) = \pi_1(\mathrm{Conf}_n(S))$

$B_n(S^2) = B_n(\mathbb{R}^2)/\Omega$



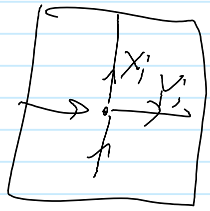
Ω

$B_n(\text{annulus}) =$



$= (\bigoplus \mathbb{Z} x_i) \tilde{\times} B_n(T^S)$

$B_n(T^2 \setminus pt) = \langle x_i, y_i, T_i \rangle / \text{rels}$



main rel = ...

$B_n(T^2) = B_n(T^2 \setminus pt) / \langle \prod x_i \text{ is central} \rangle$

$(\Leftrightarrow \prod y_i \text{ is central})$

Q:

$B_n(\mathbb{R}^2) \longleftrightarrow U_q(\mathfrak{g})$

$B_n(S) \longleftrightarrow ?$

$B_n(\text{Annulus}) \longleftrightarrow \mathcal{O}_q(G/G)$-modules

(Majid, Donin–Mudrov
reflection eq^n algebras)

$B_n(T^2 \setminus pt) \longleftrightarrow \mathcal{D}_q(G)$-mod } J.,

$B_n(T^2) \longleftrightarrow \mathcal{D}_q(G/G)$-mod } J.–Brochier

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plan: I. Clarify LHS.

II "factorization homology"

III (jt w/ Brochier, Ben-Zvi):

4D-TFT Quantum Geom. Lang. TFT

$S \longmapsto$ right hand column.

To Majid