Computing the Euler form

April-20-14 3:34 PM

$$\gamma: \begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} ax^2+2bxy+cy^2 \\ dx^2+2exy+fy^2 \end{pmatrix} = \begin{pmatrix} x \\ y \\ \phi \\ \psi \end{pmatrix}$$

$$w = \frac{t\,dx\,dy\,dz - z\,dx\,dy\,dt + y\,dx\,dz\,dt - x\,dy\,dz\,dt}{(x^2+y^2+z^2+t^2)^2}$$

$$\theta \mapsto \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\Phi: \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \longmapsto \begin{pmatrix} \cos\theta \\ \sin\theta \\ \phi_x\cos\theta + \phi_y \sin\theta \\ \psi_x\cos\theta + \psi_y \sin\theta \end{pmatrix}$$

$$\Phi^* dx = \cos\theta\, d\theta \qquad \Phi^* dy = \sin\theta\, d\theta$$

$$\Phi^* dz = (\phi_{xx}\cos\theta + \phi_{xy}\sin\theta)dx + (\phi_{xy}\cos\theta + \phi_{yy}\sin\theta)\,dy + (-\phi_x\sin\theta + \phi_y\cos\theta)\,d\theta$$

$$\Phi^* dt = (\psi_{xx}\cos\theta + \psi_{xy}\sin\theta)dx + (\psi_{xy}\cos\theta + \psi_{yy}\sin\theta)\,dy + (-\psi_x\sin\theta + \psi_x\cos\theta)\,d\theta$$