Pensieve Header: Cheat sheat β verification program, continues bbCalculus.nb and other notebooks, continued pensieve://2014-05/.

```
<< KnotTheory`
```

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Loading KnotTheory` version of April 3, 2014, 16:23:56.0784.
Read more at http://katlas.org/wiki/KnotTheory.
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```
βSimplify = Simplify;
SetAttributes[βCollect, Listable];
βCollect[B[ω_, σ_, μ_]] := B[
   βSimplify[ω], σ,
   Collect[μ, _h, Collect[#, _t, βSimplify] &]
  ];
hL[b_] := Union[Cases[b, h[s_] :> s, Infinity]];
tL[b_] := Union[Cases[b, t[s_] | T_s_ :> s, Infinity]];
dL[b_] := Union[hL[b], tL[b]];
σ_ ⊢ h_ := (∂_h σ /. 0 → 1);
βForm[B[ω_, σ_, μ_]] := Module[
   {tails, heads, mat},
   tails = tL[B[ω, σ, μ]]; heads = hL[B[ω, σ, μ]];
   mat = Outer[βSimplify[∂_{h[#1],t[#2]} μ] &, heads, tails];
   PrependTo[mat, t /@ tails];
   mat = Join[
     {Prepend[h /@ heads, ω]},
     Transpose[mat],
     {Prepend[(σ ⊢ h[#]) & /@ heads, "1+Σ/ω"]}
    ];
   MatrixForm[mat]
  ];
βForm[else_] := else /. b_B :> βForm[b];
Format[b_B, StandardForm] := βForm[b];
B /: B[ω1_, σ1_, μ1_] == B[ω2_, σ2_, μ2_] := Module[
   {heads, tails},
   tails = tL[{B[ω1, σ1, μ1], B[ω2, σ2, μ2]}];
   heads = hL[{B[ω1, σ1, μ1], B[ω2, σ2, μ2]}];
   (ω1 == ω2) && (σ1 == σ2) && (
     And @@ Flatten[Outer[
        (Coefficient[μ1, t[#1] h[#2]] == Coefficient[μ2, t[#1] h[#2]]) &,
        tails, heads
       ]]
    )
  ];

B /: B[ω1_, σ1_, μ1_] B[ω2_, σ2_, μ2_] := B[ω1 * ω2, σ1 + σ2, ω2 μ1 + ω1 μ2];
tm[x_, y_, z_][b_] := b /. {t[x] → t[z], t[y] → t[z], T_x → T_z, T_y → T_z};
hm[x_, y_, z_][B[ω_, σ_, μ_]] := B[ω,
    h[z] (σ ⊢ h[x]) (σ ⊢ h[y]) + (σ /. h[x] | h[y] → 0),
    h[z] (D[μ, h[x]] + (σ ⊢ h[x]) ∂_{h[y]} μ) + (μ /. h[x] | h[y] → 0)
   ] // βCollect;
```

```
swapth[y_, x_][B[ω_, σ_, μ_]] := Module[
   {α, β, γ, δ},
   (α β; γ δ) = (Coefficient[μ, t[y] h[x]]  D[μ, t[y]] /. h[x] → 0;
                 D[μ, h[x]] /. t[y] → 0      μ /. h[x] | t[y] → 0);
   B[ω + α, σ, {(σ ⊢ h[x]) t[y], 1}.(α  β; γ  ((ω + α) δ - γ * β) / ω).{h[x], 1}] // βCollect
  ];
dm0[x_, y_, z_][b_] := b // swapth[x, y] // hm[x, y, z] // tm[x, y, z];
dm[a_, b_, c_][B[ω0_, σ_, μ_]] := Module[
  {ω, α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ, σa, σb},
  ω = ω0 /. {T_a → T_c, T_b → T_c};
  {σa, σb} = {σ ⊢ h[a], σ ⊢ h[b]} /. {T_a → T_c, T_b → T_c};
  (α β θ; γ δ ϵ; ϕ ψ Ξ) =
   (∂_{t[a],h[a]} μ                          ∂_{t[a],h[b]} μ                          ∂_{t[a]} μ /. h[a] | h[b] → 0;
    ∂_{t[b],h[a]} μ                          ∂_{t[b],h[b]} μ                          ∂_{t[b]} μ /. h[a] | h[b] → 0;
    ∂_{h[a]} μ /. t[a] | t[b] → 0   ∂_{h[b]} μ /. t[a] | t[b] → 0   μ /. t[a] | t[b] | h[a] | h[b] →
   ) /. {T_a → T_c, T_b → T_c};
  B[ω + β,
    h[c] σa σb + (σ /. h[a] | h[b] → 0 /. {T_a → T_c, T_b → T_c}),
    {t[c], 1}.(γ + σa δ + σb (α + σa β) + (β γ - α δ)/ω   ϵ + σb θ + (β ϵ - δ θ)/ω;
               ϕ + σa ψ + (β ϕ - α ψ)/ω                   Ξ + (β Ξ - ψ θ)/ω).{h[c], 1}
    ] // βCollect
 ]

Unprotect[NonCommutativeMultiply];
b1_B ** b2_B := Module[
   {ρ, σ, labels},
   ρ = b1 * (b2 /. {h[s_] :→ h[σ[s]], t[s_] :→ t[σ[s]], T_s_ :→ T_σ[s]});
   labels = dL[{b1, b2}];
   Do[ρ = ρ // dm[s, σ[s], s], {s, labels}];
   ρ
  ];

βbRp[x_, y_] := B[1, T_x h[y], (T_x - 1) * t[x] h[y]];
βbRm[x_, y_] := B[1, h[y] / T_x, (1 / T_x - 1) * t[x] h[y]];

βbZ[L_] := Module[{s, Z, c, k},
  s = Skeleton[L];
  Z = Times @@ PD[L] /.
    X[i_, j_, k_, l_] :→ If[PositiveQ[X[i, j, k, l]], βbRp[l, i], βbRm[j, i]];
```

```
Do[Z = Z // dm[s〚c, 1〛, s〚c, k〛, s〚c, 1〛], {c, Length[s]},
 {k, 2, Length[s〚c〛]}];
Z]
```

R3 for β-better

```
{βbRp[1, 2], βbRm[1, 2]}
```

$$\left\{\begin{pmatrix} 1 & h[2] \\ t[1] & -1+T_1 \\ 1+\Sigma/\omega & T_1 \end{pmatrix}, \begin{pmatrix} 1 & h[2] \\ t[1] & -1+\frac{1}{T_1} \\ 1+\Sigma/\omega & \frac{1}{T_1} \end{pmatrix}\right\}$$

```
βbRp[1, 2] ** βbRp[1, 3]
```

$$\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & -1+T_1 & -1+T_1 \\ 1+\Sigma/\omega & 1 & T_1 & T_1 \end{pmatrix}$$

```
{βbRp[1, 2] ** βbRp[1, 3] ** βbRp[2, 3], βbRp[2, 3] ** βbRp[1, 3] ** βbRp[1, 2]}
```

$$\left\{\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & -1+T_1 & -1+T_1 \\ t[2] & 0 & 0 & T_1(-1+T_2) \\ 1+\Sigma/\omega & 1 & T_1 & T_1 T_2 \end{pmatrix}, \begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & -1+T_1 & -1+T_1 \\ t[2] & 0 & 0 & T_1(-1+T_2) \\ 1+\Sigma/\omega & 1 & T_1 & T_1 T_2 \end{pmatrix}\right\}$$

dm for β-better

```
{B0 =
  B[ω, {σa, σb, σ}.{h@a, h@b, h@S}, {t@a, t@b, t@S}.(α β θ; γ δ ϵ; ϕ ψ Ξ).{h@a, h@b, h@S}],
 B0 // dm[a, b, c]}
```

$$\left\{\begin{pmatrix} \omega & h[a] & h[b] & h[S] \\ t[a] & \alpha & \beta & \theta \\ t[b] & \gamma & \delta & \epsilon \\ t[S] & \phi & \psi & \Xi \\ 1+\Sigma/\omega & \sigma_a & \sigma_b & \sigma \end{pmatrix}, \begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{pmatrix}\right\}$$

```
(B0 // swapth[a, b] // hm[a, b, c] // tm[a, b, c]) == (B0 // dm[a, b, c]) // Simplify
True
```

$$\begin{pmatrix} \beta+\omega & \mathtt{h[c]} & \mathtt{h[S]} \\ \mathtt{t[c]} & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ \mathtt{t[S]} & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ \text{"}1+\Sigma/\omega\text{"} & \sigma_a\sigma_b & \sigma \end{pmatrix} \mathtt{\ //\ FullSimplify\ //\ MatrixForm}$$

$$\begin{pmatrix} \beta+\omega & \mathtt{h[c]} & \mathtt{h[S]} \\ \mathtt{t[c]} & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{-\delta\theta+\epsilon(\beta+\omega)+\theta\omega\sigma_b}{\omega} \\ \mathtt{t[S]} & \frac{-\alpha\psi+\phi(\beta+\omega)+\psi\omega\sigma_a}{\omega} & \Xi+\frac{\beta\Xi-\theta\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{pmatrix}$$

Back to β (and β-Burau)

$$\left((\omega+\beta)^{-1}\begin{pmatrix} \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \end{pmatrix} \mathtt{\ /.\ } \mathtt{Thread}[\{\alpha,\beta,\gamma,\delta,\theta,\epsilon,\phi,\psi,\Xi\}\to\omega\{\alpha,\beta,\gamma,\delta,\theta,\epsilon,\phi,\psi,\Xi\}]\right) \mathtt{\ //\ FullSimplify\ //\ MatrixForm}$$

$$\begin{pmatrix} \frac{\gamma+\beta\gamma-\alpha\delta+\alpha\sigma_b+\sigma_a(\delta+\beta\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\epsilon-\delta\theta+\theta\sigma_b}{1+\beta} \\ \frac{\phi+\beta\phi-\alpha\psi+\psi\sigma_a}{1+\beta} & \Xi-\frac{\theta\psi}{1+\beta} \end{pmatrix}$$

$$\mathtt{Plus}\left[\begin{pmatrix} \frac{\gamma+\beta\gamma-\alpha\delta+\alpha\sigma_b+\sigma_a(\delta+\beta\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\epsilon-\delta\theta+\theta\sigma_b}{1+\beta} \\ \frac{\phi+\beta\phi-\alpha\psi+\psi\sigma_a}{1+\beta} & \Xi-\frac{\theta\psi}{1+\beta} \end{pmatrix} \mathtt{\ /.\ } \{\alpha\to\alpha+\sigma_a,\ \delta\to\delta+\sigma_b,\ \Xi\to\Xi+\sigma\},\ \begin{pmatrix} -\sigma_a\sigma_b & 0 \\ 0 & -\sigma \end{pmatrix}\right] \mathtt{\ //\ FullSimplify\ //\ MatrixForm}$$

$$\begin{pmatrix} \gamma-\frac{\alpha\delta}{1+\beta} & \epsilon-\frac{\delta\theta}{1+\beta} \\ \phi-\frac{\alpha\psi}{1+\beta} & \Xi-\frac{\theta\psi}{1+\beta} \end{pmatrix}$$

$$(1+\beta)\begin{pmatrix} \gamma-\frac{\alpha\delta}{1+\beta} & \epsilon-\frac{\delta\theta}{1+\beta} \\ \phi-\frac{\alpha\psi}{1+\beta} & \Xi-\frac{\theta\psi}{1+\beta} \end{pmatrix} \mathtt{\ //\ FullSimplify\ //\ MatrixForm}$$

$$\begin{pmatrix} \gamma+\beta\gamma-\alpha\delta & \epsilon+\beta\epsilon-\delta\theta \\ \phi+\beta\phi-\alpha\psi & \Xi+\beta\Xi-\theta\psi \end{pmatrix}$$

$$\text{Plus}\left[\begin{pmatrix} \frac{\gamma+\beta\,\gamma-\alpha\,\delta+\alpha\,\sigma_b+\sigma_a\,(\delta+\beta\,\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\,\epsilon-\delta\,\Theta+\Theta\,\sigma_b}{1+\beta} \\ \frac{\phi+\beta\,\phi-\alpha\,\psi+\psi\,\sigma_a}{1+\beta} & \Xi-\frac{\Theta\,\psi}{1+\beta} \end{pmatrix} \text{/. }\{\alpha\to\alpha+\sigma_a-1,\ \delta\to\delta+\sigma_b-1,\ \Xi\to\Xi+\sigma-1\},\ \begin{pmatrix} 1-\sigma_a\,\sigma_b & 0 \\ 0 & 1-\sigma \end{pmatrix}\right] \text{ // FullSimplify // MatrixForm}$$

$$\begin{pmatrix} \frac{\alpha+\beta+\gamma+\beta\,\gamma+\delta-\alpha\,\delta}{1+\beta} & \in+\frac{\Theta-\delta\,\Theta}{1+\beta} \\ \phi+\frac{\psi-\alpha\,\psi}{1+\beta} & \Xi-\frac{\Theta\,\psi}{1+\beta} \end{pmatrix}$$

$$\text{Plus}\left[\begin{pmatrix} \frac{\gamma+\beta\,\gamma-\alpha\,\delta+\alpha\,\sigma_b+\sigma_a\,(\delta+\beta\,\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\,\epsilon-\delta\,\Theta+\Theta\,\sigma_b}{1+\beta} \\ \frac{\phi+\beta\,\phi-\alpha\,\psi+\psi\,\sigma_a}{1+\beta} & \Xi-\frac{\Theta\,\psi}{1+\beta} \end{pmatrix} \text{/. }\{\alpha\to\alpha-1,\ \delta\to\delta-1,\ \Xi\to\Xi-1\},\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] \text{ // FullSimplify // MatrixForm}$$

$$\begin{pmatrix} \frac{\alpha+\beta+\gamma+\beta\,\gamma+\delta-\alpha\,\delta+(-1+\alpha)\,\sigma_b+\sigma_a\,(-1+\delta+\beta\,\sigma_b)}{1+\beta} & \frac{\in+\beta\,\in+\Theta-\delta\,\Theta+\Theta\,\sigma_b}{1+\beta} \\ \frac{\phi+\beta\,\phi+\psi-\alpha\,\psi+\psi\,\sigma_a}{1+\beta} & \Xi-\frac{\Theta\,\psi}{1+\beta} \end{pmatrix}$$

Alexander with β-better

```
{Knot[8, 17] // βbZ, Alexander[Knot[8, 17]][T1] // βSimplify}
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\left\{\begin{pmatrix} -8-\frac{1}{T_1^2}+\frac{4}{T_1}+11\,T_1-8\,T_1^2+4\,T_1^3-T_1^4 & h[1] \\ t[1] & 0 \\ 1+\Sigma/\omega & 1 \end{pmatrix},\ 11-\frac{1}{T_1^3}+\frac{4}{T_1^2}-\frac{8}{T_1}-8\,T_1+4\,T_1^2-T_1^3\right\}$$

The MVA with β-better

```
βbMVA[L_Link] := Module[{Hs, ω, σ, μ, A, M},
  {ω, σ, μ} = List @@ βbZ[L];
  Hs = Rest[h /@ (First /@ Skeleton[L])];
  A = Outer[Coefficient[μ, #1 * #2] &, Hs, Hs /. h[a_] :→ t[a]];
  M = A - ω DiagonalMatrix[((σ ⊢ #) - 1) & /@ Hs];
  Factor[ω^(2-Length@Skeleton@L) Det[M] / (1 - T_Skeleton[L][[1,1]])]
 ]
```

```
βbMVA[Link["L6a4"]]
```

$$-\frac{(-1+T_1)\,(-1+T_5)\,(-1+T_9)}{T_1\,T_5}$$

```
Factor[1/(βbMVA[#]) (MultivariableAlexander[#][T] /. T[i_] :> T_{Skeleton[#][[i,1]]})] & /@
 AllLinks[{2, 8}]
```

KnotTheory::loading : Loading precomputed data in MultivariableAlexander4Links`.

$$\left\{T_1^2 T_3, T_1^{3/2} T_5^{3/2}, \sqrt{T_1}\, T_5^{3/2}, T_1^{3/2} \sqrt{T_5}, T_1^2 T_7^2, T_1^2 T_7^2, -\frac{\sqrt{T_1}\sqrt{T_5}}{\sqrt{T_9}}, -T_1^{3/2} T_5^{3/2} T_9^{3/2}, -\frac{\sqrt{T_1}\sqrt{T_5}}{T_9^{3/2}}, \sqrt{T_1}\sqrt{T_5}, T_1^{3/2} T_5^{7/2}, \frac{\sqrt{T_1}}{T_5^{3/2}}, \frac{\sqrt{T_1}}{T_5^{3/2}}, T_1 T_7^2, \frac{1}{T_7}, -\frac{T_1^{3/2}\sqrt{T_5}}{\sqrt{T_9}}, T_1^{3/2} T_5^{7/2}, \sqrt{T_1}\, T_5^{5/2}, \sqrt{T_1}\, T_5^{3/2}, \frac{\sqrt{T_1}}{\sqrt{T_5}}, T_1^{3/2} T_5^{3/2}, \sqrt{T_1}\, T_5^{3/2}, \frac{T_1^{3/2}}{\sqrt{T_5}}, \frac{T_1^{3/2}}{\sqrt{T_5}}, T_1^{3/2} T_5^{7/2}, \frac{T_1}{T_7}, T_1 T_7, T_1^2 T_7^3, T_1^2 T_7^3, T_1^{5/2} T_9^{5/2}, T_1^{5/2} T_9^{5/2}, T_1^{5/2} T_9^{5/2}, -T_1^{3/2} T_5^{3/2}\sqrt{T_9}, -\sqrt{T_1}, -T_1^{3/2} T_5^2 T_{11}^2, -\frac{\sqrt{T_1}}{T_{11}^2}, -\frac{\sqrt{T_1}\, T_5}{T_{11}}, -\frac{T_1^{3/2}\sqrt{T_5}}{T_{13}^{3/2}}, T_1^{3/2} T_5^{3/2} T_9^{3/2} T_{13}^{3/2}, T_1^{3/2} T_5^{3/2}, \sqrt{T_1}\sqrt{T_5}, -T_1^{3/2} T_5^2 T_{11}^2, -\sqrt{T_1}\, T_5^2 T_{11}, -\sqrt{T_1}\, T_5^{3/2} T_{11}^{3/2}, -T_1^{3/2} T_5^{5/2}\sqrt{T_{13}}, \frac{\sqrt{T_1}\sqrt{T_5}}{\sqrt{T_9}\sqrt{T_{13}}}, \sqrt{T_1}\sqrt{T_5}\sqrt{T_9}\sqrt{T_{13}}\right\}$$

Burau Calculus

```
Plus[
  ( β+ω        h[c]                                  h[S]
    t[c]       γ + (βγ-αδ)/ω + δ σa + (α+β σa) σb     (βϵ-δθ+ϵω+θωσb)/ω
    t[S]       (βϕ-αψ+ϕω+ψωσa)/ω                      (βΞ-θψ+Ξω)/ω
    "1+Σ/ω"    σa σb                                  σ ) /.
  {α → α + ω σa, δ → δ + ω σb, Ξ → Ξ + ω σ},
  - ( 0  0        0
      0  ω σa σb  0
      0  0        ω σ
      0  0        0 )
 ] // FullSimplify // MatrixForm
```

$$\begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \frac{-\alpha\,\delta+\gamma\,(\beta+\omega)+\beta\,\omega\,\sigma_a\,\sigma_b}{\omega} & \epsilon+\frac{\beta\,\epsilon-\delta\,\theta}{\omega} \\ t[S] & \phi+\frac{\beta\,\phi-\alpha\,\psi}{\omega} & \Xi+\beta\,\sigma+\frac{\beta\,\Xi-\theta\,\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\,\sigma_b & \sigma \end{pmatrix}$$

$$\text{Plus}\Big[\begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ \text{"}1+\Sigma/\omega\text{"} & \sigma_a\sigma_b & \sigma \end{pmatrix} /. \{\alpha\to\alpha+\omega(\sigma_a-1),\ \delta\to\delta+\omega(\sigma_b-1),\ \Xi\to\Xi+\omega(\sigma-1)\},$$
$$-\begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega(\sigma_a\sigma_b-1) & 0 \\ 0 & 0 & \omega(\sigma-1) \\ 0 & 0 & 0 \end{pmatrix}\Big] \text{ // FullSimplify // MatrixForm}$$

$$\begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \alpha+\gamma+\delta+\frac{\beta\gamma-\alpha\delta}{\omega}+\beta\sigma_a\sigma_b & \epsilon+\theta+\frac{\beta\epsilon-\delta\theta}{\omega} \\ t[S] & \phi+\psi+\frac{\beta\phi-\alpha\psi}{\omega} & \Xi+\beta(-1+\sigma)+\frac{\beta\Xi-\theta\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{pmatrix}$$

$$\text{Plus}\Big[\begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ \text{"}1+\Sigma/\omega\text{"} & \sigma_a\sigma_b & \sigma \end{pmatrix} /. \{\alpha\to\alpha-\omega,\ \delta\to\delta-\omega,\ \Xi\to\Xi-\omega\},$$
$$+\begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \\ 0 & 0 & 0 \end{pmatrix}\Big] \text{ // FullSimplify // MatrixForm}$$

$$\begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \frac{\beta\gamma-\alpha\delta+(\alpha+\gamma+\delta)\omega+(\alpha-\omega)\omega\sigma_b+\omega\sigma_a(\delta-\omega+\beta\sigma_b)}{\omega} & \epsilon+\theta+\frac{\beta\epsilon-\delta\theta}{\omega}+\theta\sigma_b \\ t[S] & \phi+\psi+\frac{\beta\phi-\alpha\psi}{\omega}+\psi\sigma_a & -\beta+\Xi+\frac{\beta\Xi-\theta\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{pmatrix}$$