Minsky@Blyth: Complexity in surfaces and 3-manifolds, 3

March-12-14 4:03 PM

j/ w/ Namazi, Brock, Souto.

Today: cutting along compressible surfaces.

Motivation: $M = H^+ \cup H^-$ "Heegaard splitting"

$H^\pm$: handlebodies of genus g ← very compressible!

Good news: Heegaard splittings always exist
Bad news: There are too many of them.

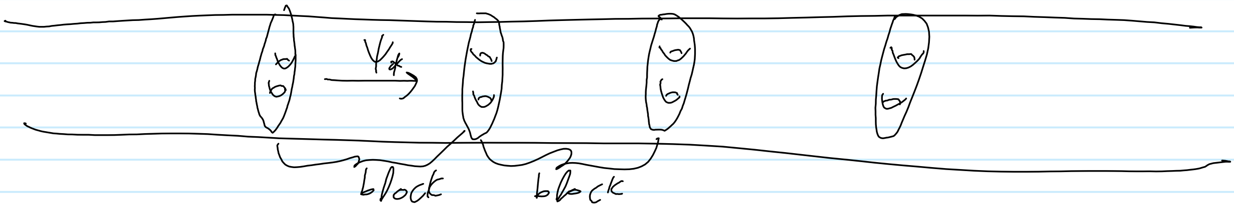
Fix genus g, pick gluing map $\psi: \partial H^+ \to \partial H^-$.

<u>Def</u> $\psi: \Sigma \to \Sigma$ is pseudo-Anosov (ψA) if it is irreducible: fixes no finite collection of isotopy classes of curves.

<u>Thm</u> $M_\psi := \Sigma \times [0,1] \Big/ (x,0) \sim (\psi(x),1)$ is hyperbolic iff ψ is pseudo-Anosov

Let $\hat{M}_\psi$ be the $\underbrace{\text{the}}_{\text{obvious}}$ $\mathbb{Z}$-cover of M_ψ.

Given ψA, it is also hyperbolic.



<u>Thm</u> Let $H^\pm, \Sigma$ be fixed, $\psi: \Sigma \circlearrowleft$ be generic (needs definition) ψA. Let $N_m = H^+ \cup_{\psi^m} H^-$.

For large m, N_m is hyperbolic.

Moreover, N_m has a "model metric"

$N_m, \overset{\text{metric}}{\mu}$

which is k-biliph isomorphic to the hyperbolic metric. (uniformly in m)

<u>Thurston's double-limit Theorem</u>. Let Σ_g be fixed,

$$\rho_n : \pi_1 \Sigma_g \longrightarrow PSL(2,\mathbb{C})$$

discrete, faithful, $\exists$ curves α_n, β_n on Σ_g

isotopy classes of simple closed

s.t. $\quad * \ \ell(\rho_n(\alpha_n)), \ell(\rho_n(\beta_n))$ bounded.

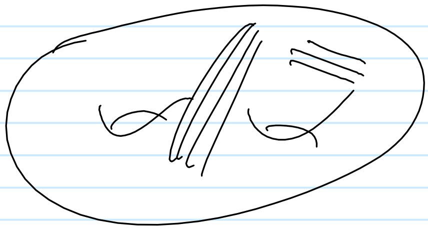
$* \ \alpha_n \longrightarrow \lambda$ a lamination on Σ

$\beta_n \longrightarrow \mu$ — 11 —

such that $\lambda \cup \mu$ binds Σ: $\Sigma \setminus (\lambda \cup \mu)$ is a union of disks.

<u>Then</u> ρ_n has a subsequence that converges up to conjugacy in $PSL(2,\mathbb{C})$

<u>Laminations</u>:



closed subset foliated by geodesics.