Alexander Braverman: Instanton Moduli Spaces and w-Algebras

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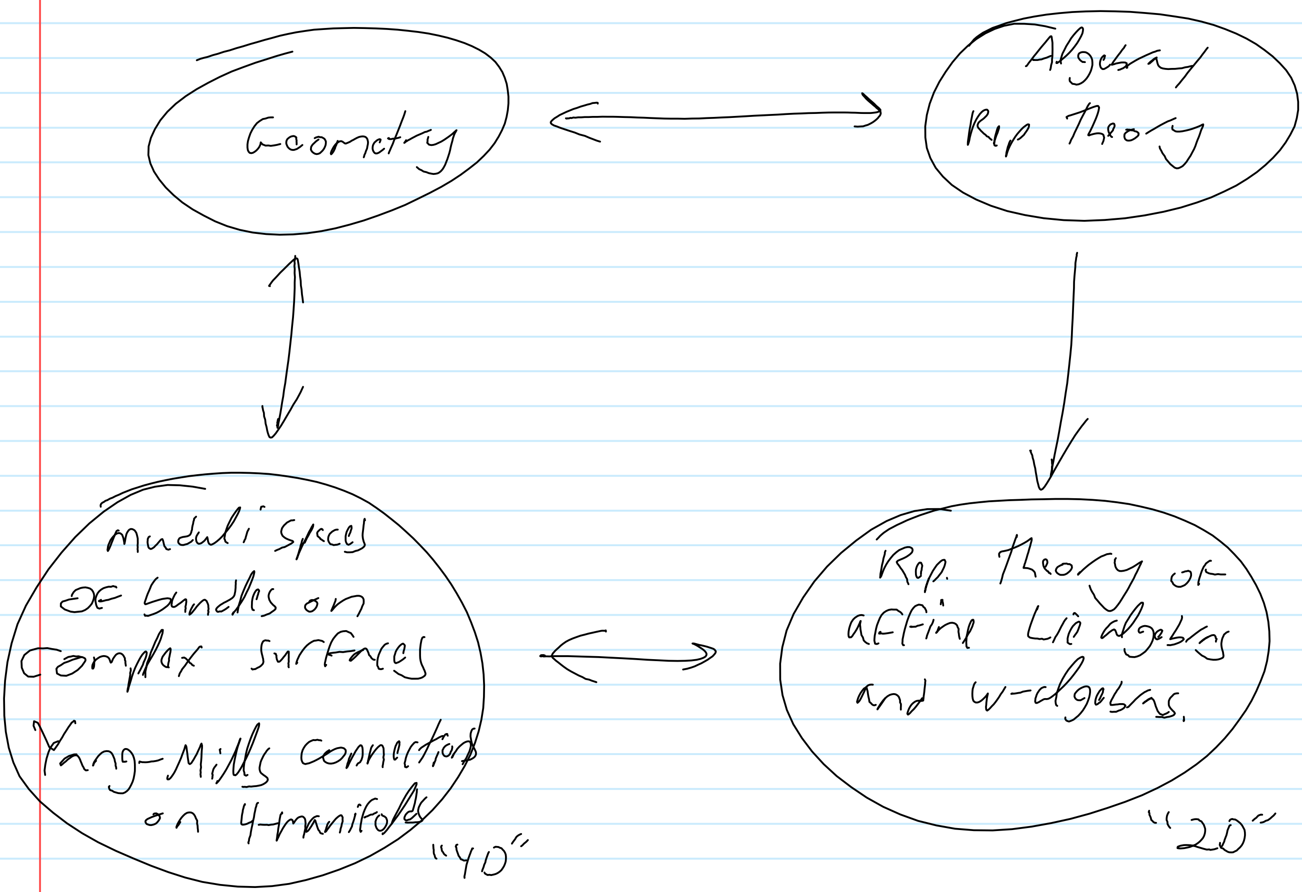
Preview of the talk j/w/ Finkelberg, Nakajima



Motivation

1. SUSY QFT "Dualities" between 4D theories & 2D theories (CFTs)

   Alday-Gaioto-Tach.. (AG) conjecture implies work here.

2. Corollary of geometric Langlands for affine Kac-Moody groups.

How do group representations arise in "nature"?

(1) Groups of symmetries, act on associated vector spaces.

(2) Example of 2nd cause: "Hodge Theoretic $sl(2)$".

The subject of the talk is to produce something like that in other contexts.

Geometric side —

G: simple complex group.

Moduli space of G-bundles on $\mathbb{C}^2$

"principal holomorphic"

$Bun_G(\mathbb{C}^2) = \{G\text{-bundles on } \mathbb{CP}^2 \text{ trivialized at } \mathbb{CP}^1\}$

$Bun_G^d(\mathbb{C}^2) = \ldots$ w/ $c_2 = d$.

It is a f.d. complex manifold of dim. $2d h_G^\vee$ $h_G^\vee$: dual coxeter number.

$Bun_G^d(\mathbb{C}^2)$ has an action of $G \times GL(2,\mathbb{C})$

changes triv. at ∞

acts on $\mathbb{C}^2$

$Bun_G^d \subset U_G^d$ — Uhlenbeck space

Main Geometric object: A certain cohomology (intersection, equivariant) of U_G^d

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Algebraic Side: Want "affine W-algebras"

$\mathfrak{g}$ - simple Lie algebra / $\mathbb{C}$

$U(\mathfrak{g})$ = univ. env. alg.

$Z(\mathfrak{g})$: Center of $U(\mathfrak{g})$

Harish-Chandra isomorphism: $Z(\mathfrak{g}) \cong \mathrm{Sym}(\mathfrak{h})^W$

$\mathfrak{g} \supset \mathfrak{h}$ cartan W: Weyl group.

Infinite dim algebras:

$$0 \to \underset{\mathbb{1}}{\mathbb{C}} \to \hat{\mathfrak{g}} \to \mathfrak{g}((t)) \to 0$$

affine Lie algebra

$U_k(\hat{\mathfrak{g}}) :=$ $\quad k \in \mathbb{C}$

$U(\hat{\mathfrak{g}}) / \mathbb{1} = k$

Feigin-Frenkel 1. for generic k, no center

2. at $k = -h^\vee$, $\qquad \mathrm{Sym}(L((t)))$

"the critical level"

3. $Z_{-h^\vee}$ has a natural n.c. deformation

$W_k(\mathfrak{g})$ "W-algebra"

A. $H^*_G(X) =$ "$H^*(X/G)$"

(if $G \circlearrowright X$ freely)

$$H^*_G(pt) = \mathrm{Sym}(\mathfrak{h}^*)^W$$

$H^*_G(X)$ is always a module over $H^*_G(pt)$

B. Intersection / Goresky-Macpherson cohomology

$IH^*(X)$

There's also $IH^*_G(X)$

<u>Main Thm</u> (F, N, B) $\exists$ action of

$W_k(\mathfrak{g})$ on $\bigoplus_{d \geq 0} IH^*_{G \times GL(2)}(\mathcal{U}^d_G)$

and all structures on both sides match.