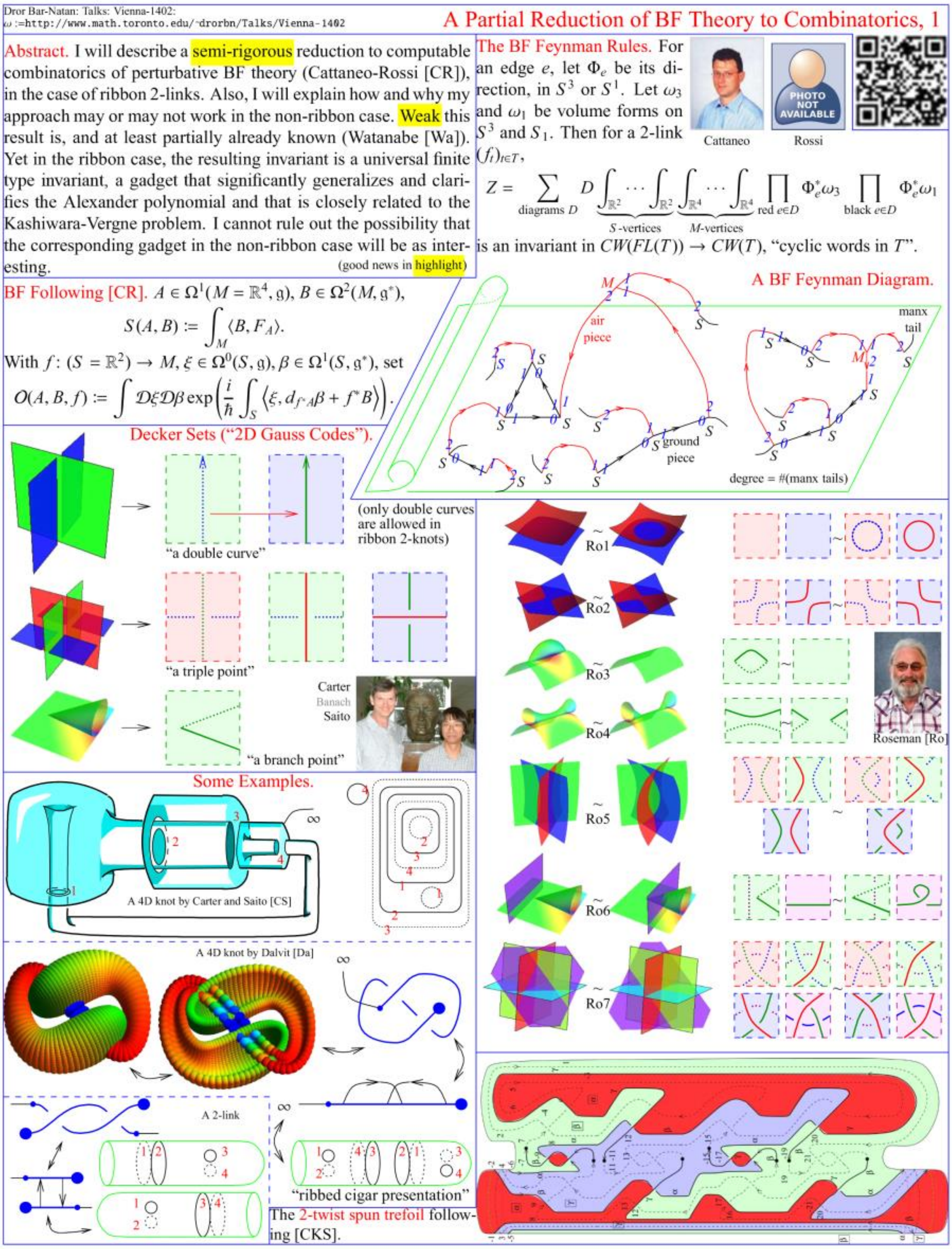
Vienna-1402 Handout on February 8, 2014

February-08-14 10:30 AM



Dror Bar-Natan: Talks: Vienna-1402:
ω :=http://www.math.toronto.edu/~drorbn/Talks/Vienna-1402

A Partial Reduction of BF Theory to Combinatorics, 1

Abstract. I will describe a semi-rigorous reduction to computable combinatorics of perturbative BF theory (Cattaneo-Rossi [CR]), in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. Weak this result is, and at least partially already known (Watanabe [Wa]). Yet in the ribbon case, the resulting invariant is a universal finite type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Kashiwara-Vergne problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be as interesting. (good news in highlight)

The BF Feynman Rules. For an edge e, let Φ_e be its direction, in S^3 or S^1. Let ω_3 and ω_1 be volume forms on S^3 and S_1. Then for a 2-link $(f_t)_{t\in T}$,

$$Z=\sum_{\text{diagrams }D} D\underbrace{\int_{\mathbb{R}^2}\cdots\int_{\mathbb{R}^2}}_{S\text{-vertices}}\underbrace{\int_{\mathbb{R}^4}\cdots\int_{\mathbb{R}^4}}_{M\text{-vertices}}\prod_{\text{red } e\in D}\Phi_e^*\omega_3\prod_{\text{black } e\in D}\Phi_e^*\omega_1$$

is an invariant in $CW(FL(T))\to CW(T)$, "cyclic words in T".

Cattaneo Rossi

BF Following [CR]. $A\in\Omega^1(M=\mathbb{R}^4,\mathfrak{g})$, $B\in\Omega^2(M,\mathfrak{g}^*)$,

$$S(A,B):=\int_M\langle B,F_A\rangle.$$

With $f\colon(S=\mathbb{R}^2)\to M$, $\xi\in\Omega^0(S,\mathfrak{g})$, $\beta\in\Omega^1(S,\mathfrak{g}^*)$, set

$$O(A,B,f):=\int\mathcal{D}\xi\mathcal{D}\beta\exp\left(\frac{i}{\hbar}\int_S\left\langle\xi,d_{f^*A}\beta+f^*B\right\rangle\right).$$

A BF Feynman Diagram. air piece; ground piece; manx tail; degree = #(manx tails)

Decker Sets ("2D Gauss Codes"). "a double curve" (only double curves are allowed in ribbon 2-knots) "a triple point" "a branch point"

Carter Banach Saito

Ro1 Ro2 Ro3 Ro4 Ro5 Ro6 Ro7

Roseman [Ro]

Some Examples. A 4D knot by Carter and Saito [CS]

A 4D knot by Dalvit [Da]

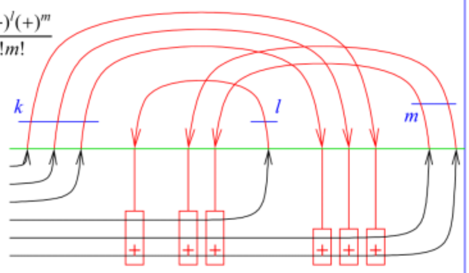
A 2-link

"ribbed cigar presentation"

The 2-twist spun trefoil following [CKS].

Theorem. Finish (also note relationship w/ KBH)

$$\sum_{k,l,m\geq 0}\frac{(+)^k(-)^l(+)^m}{k!l!m!}$$

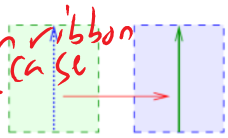


Also add a link example

In 4D Axial Gauge.

- "Drop down" red propagators.
- No trivalent M vertices.
- No need for black cycles(?).

in ribbon case



References.

[CKS] J. S. Carter, S. Kamada, and M. Saito, *Diagrammatic Computations for Quandles and Cocycle Knot Invariants,* Contemp. Math. **318** (2003) 51–74.

[CS] J. S. Carter and M. Saito, *Knotted surfaces and their diagrams,* Mathematical Surveys and Monographs **55**, American Mathematical Society, Providence 1998.

[Da] E. Dalvit, http://science.unitn.it/~dalvit/.

[CR] A. S. Cattaneo and C. A. Rossi, *Wilson Surfaces and Higher Dimensional Knot Invariants,* Commun. in Math. Phys. **256-3** (2005) 513–537, arXiv:math-ph/0210037.

[Ro] D. Roseman, *Reidemeister-Type Moves for Surfaces in Four-Dimensional Space,* Knot Theory, Banach Center Publications **42** (1998) 347–380.

[Wa] T. Watanabe, *Configuration Space Integrals for Long n-Knots, the Alexander Polynomial and Knot Space Cohomology,* Alg. and Geom. Top. **7** (2007) 47-92, arXiv:math/0609742.

Issues.

- ~~A decker set example with a triple point.~~

"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

www.katlas.org The Knot Atlas

Musings.

* why does BF restrict to CS?
* Is this all? The v-invariant.
* Something about gnots.
* what are finite-type invariants? what are

"chord diagrams"?

* Bubble-wrap-finite-type.
* "shielded 2-tangles" / Foams; make pictures of the 3D

 & 

is this related to KV?

* Something about invariants of plane curves.

Decide on a booklet!

A bubble wrap