Vienna-1402 Handout on February 5, 2014

February-05-14 5:00 PM

Dror Bar-Natan: Talks: Vienna-1402:
ω :=http://www.math.toronto.edu/~drorbn/Talks/Vienna-1402

A Partial Reduction of BF Theory to Combinatorics, 1

Abstract. I will describe a semi-rigorous reduction to computable combinatorics of perturbative BF theory (Cattaneo-Rossi [CR]), in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. Weak this result is, and at least partially already known (Watanabe [Wa]). Yet in the ribbon case, the resulting invariant is a universal finite type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Kashiwara-Vergne problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be as interesting. (good news in highlight)

BF Following [CR]. $A \in \Omega^1(M = \mathbb{R}^4, \mathfrak{g})$, $B \in \Omega^2(M, \mathfrak{g}^*)$,

$$S(A,B) := \int_M \langle B, F_A \rangle.$$

With $f\colon (S = \mathbb{R}^2) \to M$, $\xi \in \Omega^0(S, \mathfrak{g})$, $\beta \in \Omega^1(S, \mathfrak{g}^*)$, set

$$O(A,B,f) := \int \mathcal{D}\xi\mathcal{D}\beta \exp\left(\frac{i}{\hbar}\int_S \langle \xi, d_{f^*A}\beta + f^*B \rangle\right).$$

2D

The BF Feynman Rules. For an edge e, let Φ_e be its direction, in S^3 or S^1. Let ω_3 and ω_1 be volume forms on S^3 and S_1. Then for a 2-link $(f_i)_{i\in T}$,

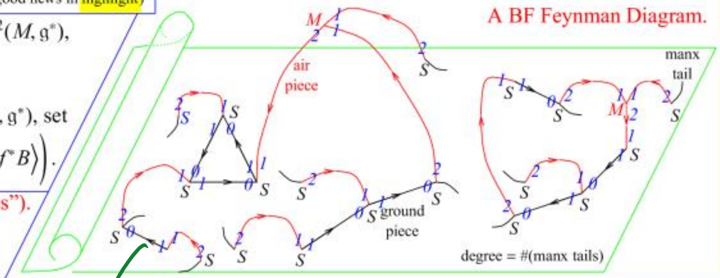
Cattaneo

Rossi

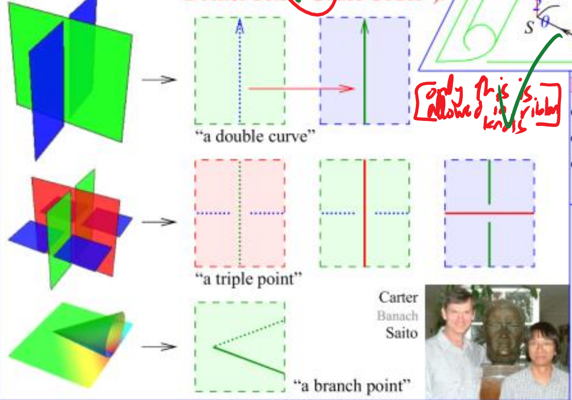
$$Z = \sum_{\text{diagrams } D} D \underbrace{\int_{\mathbb{R}^2} \cdots \int_{\mathbb{R}^2}}_{S\text{-vertices}} \underbrace{\int_{\mathbb{R}^4} \cdots \int_{\mathbb{R}^4}}_{M\text{-vertices}} \prod_{\text{red } e\in D} \Phi_e^*\omega_3 \prod_{\text{black } e\in D} \Phi_e^*\omega_1$$

is an invariant in $CW(FL(T)) \to CW(T)$, "cyclic words in T".

A BF Feynman Diagram.



Decker Sets ("Gauss Codes").

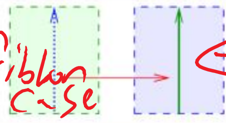


only this is allowed in ribbon knots

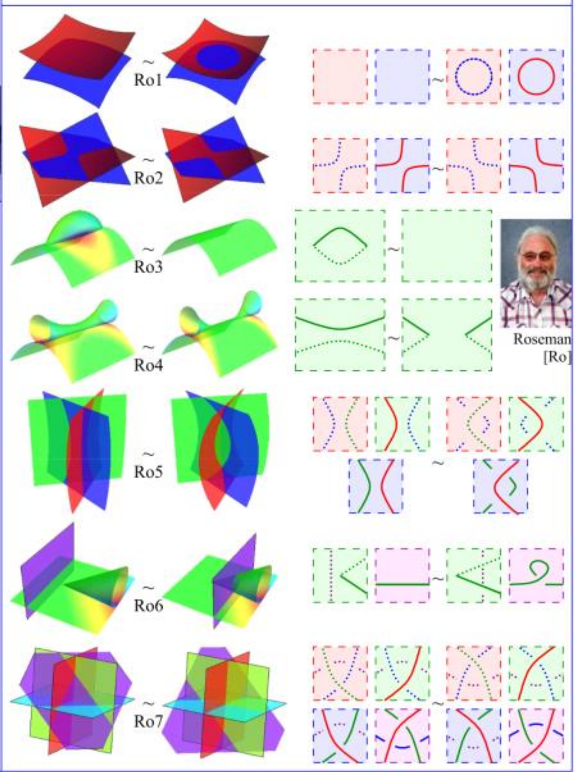
In 4D Axial Gauge.

- "Drop down" red propagators.
- No trivalent M vertices.
- No need for black cycles(?).

in ribbon case

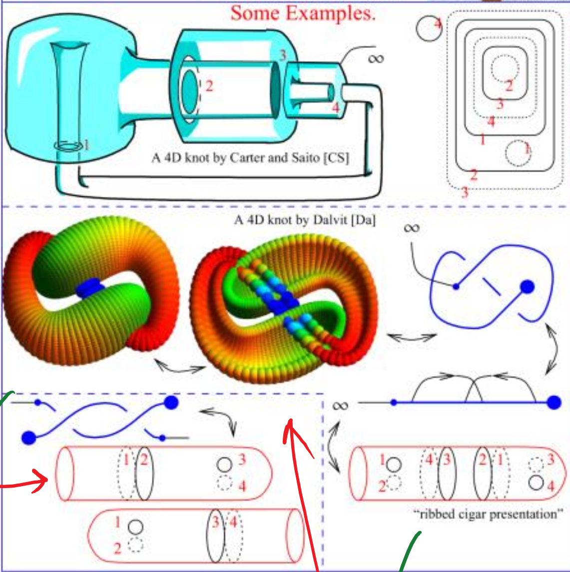


put on page 2



non-ribbon example.

Some Examples.



green

also include the 1D gauss code.

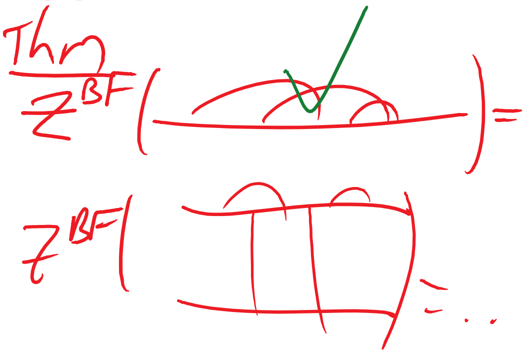
Dror Bar-Natan: Talks: Vienna-1402:
ω :=http://www.math.toronto.edu/~drorbn/Talks/Vienna-1402

A Partial Reduction of BF Theory to Combinatorics, 2

~~Axial gauge in the ribbon case.~~

~~Axial gauge in the ribbon case.~~

Thm



References.

[CS] J. S. Carter and M. Saito, *Knotted surfaces and their diagrams,* Mathematical Surveys and Monographs **55**, American Mathematical Society, Providence 1998.

[Da] E. Dalvit, `http://science.unitn.it/~dalvit/`.

[CR] A. S. Cattaneo and C. A. Rossi, *Wilson Surfaces and Higher Dimensional Knot Invariants,* Commun. in Math. Phys. **256-3** (2005) 513–537, arXiv:math-ph/0210037.

[Ro] D. Roseman, *Reidemeister-Type Moves for Surfaces in Four-Dimensional Space,* Knot Theory, Banach Center Publications **42** (1998) 347–380.

[Wa] T. Watanabe, *Configuration Space Integrals for Long n-Knots, the Alexander Polynomial and Knot Space Cohomology,* Alg. and Geom. Top. **7** (2007) 47-92, arXiv:math/0609742.

Issues.

- A decker set example with a triple point.

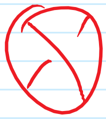
"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

www.katlas.org

Musings.

* why does BF restrict to CS?
* Is this all? The v-invariant.
* Something about gnots.
* what are finite-type invariants? what are "chord diagrams"?

* Bubble-wrap-finite-type.
* "shielded 2-tangles" / foams; make pictures of the 3D

 & 

is this related to KV?

* Something about invariants of plane curves.

Decide on a booklet!