Ulrike Tillmann on Commutative K-Theory and other new generalised cohomology theories

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G: Lie group

$BG = |B_\bullet G|$ where $B_n G = G^n$

$$\partial_i(g_1 \ldots g_n) = \begin{cases} (g_2 \ldots g_n) & i=1 \\ (g_1 \ldots g_i g_{i+1}, \ldots g_n) \\ (g_1 \ldots g_n) \end{cases}$$

$\rightsquigarrow \mathrm{Hom}(\mathbb{Z}^n, G)$

$\rightsquigarrow$ descending central series:

$\Gamma^1 = F_n \qquad \Gamma^q = [\Gamma^{q-1}, F_n]$

$B(q,G) := \mathrm{Hom}(F_n/\Gamma^q, G)$

$B(q,G) = |B_\bullet(q,G)|$

$[X, B(q,G)]$ = iso. classes of G-bundles whose transition functions can be chosen to be in subgroups of G of nilpotency class $< q$.

a bit imprecise as written

<u>Thm</u> If G is a reductive group with K its maximal compact subgrp, then $B(q,G) \simeq B(q,K)$.

<u>Thm</u> For $G = SU, U, SO, Sp, O$, $B(q,G)$ are Ω^∞-spaces giving a filtration of BG. Furthermore,

There is a splitting of Ω^∞-spaces

$$B(Q,G) \simeq E(Q,G) \times BG$$

where

$$\begin{tikzcd} E(Q,G) \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & E(G) \arrow[d] \\ B(Q,G) \arrow[r] & BG \end{tikzcd}$$

notation means "pull back". I should figure out the origin.

Ω^∞-space machine:

May–Segal: $\mathcal{C}$ symmetric monoidal $\Rightarrow$ $B\mathcal{C}$ is an E_∞-space.

$\vdots$

<u>Example</u> $\mathcal{C}$: obj: $\mathbb{C}^n$

morph: $GL_n(\mathbb{C})$ or U_n

$$B\mathcal{C} = \coprod BGL_n(\mathbb{C}) \text{ or } \coprod BU_n$$

$$\Rightarrow \Omega B(\ \ ) = BU.$$

$\vdots$