Ulrich Bunke on Differential cohomology theory

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1. Cohomology theory on manifolds

$$h^*: \{\text{manifolds}\} \longrightarrow \{\mathbb{Z} \text{ graded Abelian groups}\}$$

Such that

1. Mayer-Vietoris: $M = U \cup V$

$$h^{i-1}(U \cap V) \xrightarrow{\partial^{i-1}} h^i(M) \longrightarrow h^i(U) \cap$$

2. $h^*(\coprod_i M_i) \cong \prod_i h^*(M_i)$

3. homotopy invariance:

$$h^*(M) \xrightarrow{\sim} h^*(I \times M)$$

... Come from a spectrum $h^i(M) = \pi_0(\Sigma^i E^M)$

Examples $H\mathbb{Z}$, $H\mathbb{R}$, K

What happens if 3. is dropped?

Example $C^\infty: \begin{cases} 0 & i \neq 0 \\ C^\infty(M) & i = 0 \end{cases}$ $\partial = 0$

Example $H(\sigma^{\geq k} \Omega)$ a cutoff of de-Rham.

Example $\mathrm{Diff}^1(\mathbb{Z} \subset \mathbb{R}): \begin{cases} 0 & i \leq 0 \\ C^\infty(M, U(1)) & i = 1 \\ H^i(M, \mathbb{Z}) & i \geq 2 \end{cases}$

Example $\mathrm{Diff}^2(\mathbb{Z} \subset \mathbb{R})$: $\begin{cases} i \le 0 & 0 \\ i = 1 & (\mathbb{R}/\mathbb{Z})^M \\ i = 2 & \text{Line bundles with flat connection?} \\ i \geq 3 & H^i(M, \mathbb{Z}) \end{cases}$