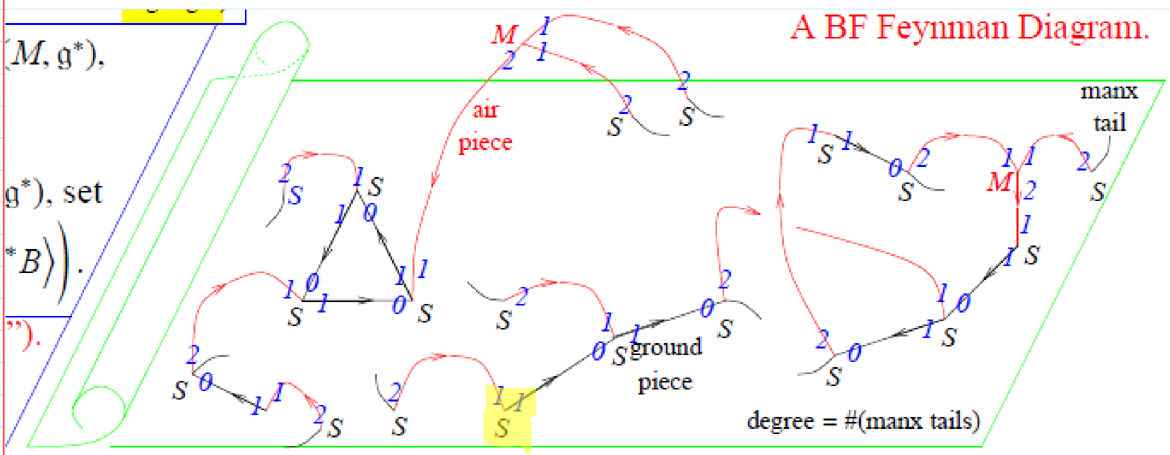
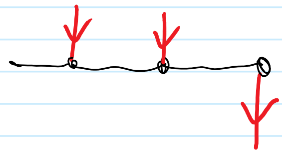
The Algebraic Structure Underlying BF

February-16-14 1:14 PM



[A, [A, B]]:



not as above!

A Lie algebra $\mathfrak{g}$.

A $\mathfrak{g}$-module B

A map $\phi: \mathfrak{g} \to B$ s.t.

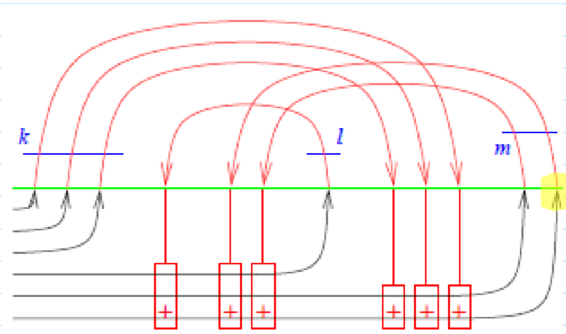
$$\phi([x,y]) = [\phi(x), y] + [x, \phi(y)]$$

Namely, a B-value 1-cocycle.

But what about the tails?

Wrong tail placements above.

Do not match!



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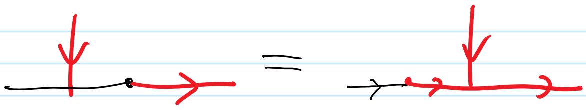

A map $\Psi: B \to \mathfrak{g}$ satisfying $\Psi(gb) = [g, \Psi(b)]$

BF Following [CR]. $A \in \Omega^1(M = \mathbb{R}^4, \mathfrak{g})$, $B \in \Omega^2(M, \mathfrak{g}^*)$,

$$S(A, B) := \int_M \langle B, F_A \rangle.$$

With $f\colon (S = \mathbb{R}^2) \to M$, $\xi \in \Omega^0(S, \mathfrak{g})$, $\beta \in \Omega^1(S, \mathfrak{g}^*)$, set

$$O(A, B, f) := \int \mathcal{D}\xi \mathcal{D}\beta \exp\left(\frac{i}{\hbar} \int_S \langle \xi, d_{f^*A}\beta + f^*B \rangle\right).$$

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And that's it.