Chris Schommer-Pries on Extended topological field theories and tensor categories arXiv:1312.7188

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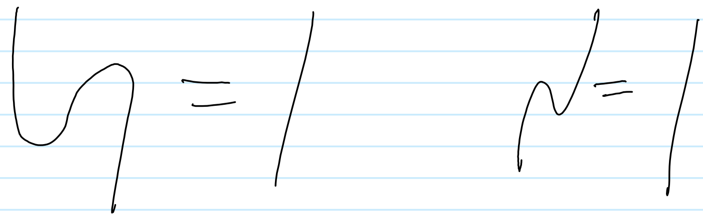
Defn A tensor category:

- Abelian, enriched over Vect
- Monoidal, $\otimes$.
- Finite:
  - Hom spaces are f.d. v.s.
  - Every object has finite length: $X \supsetneq X_1 \supsetneq X_2 \supsetneq \ldots$ such seqs are finite.
  - Enough projectives
  - Finitely many simple objects.
- Rigid: Every object has left & right dual,

  ev: $X \otimes {}^*X \longrightarrow 1$

  coev: $1 \longrightarrow {}^*X \otimes X$

  s.t.



Thm (Etingof–Nikshych–Ostrik)

For every tensor category, $X \mapsto X^{****}$ is canonically isomorphic to conjugation by some distinguished invertible object $D \in \mathcal{C}$:

$X^{****} \cong D \otimes X \otimes D^{-1}$

- an analog of "Radford's S^4 formula for Hopf algebras".

If $\mathcal{C}$ is <u>fusion</u> (semisimple & 1 is simple) then $D = 1$. I.e., $X^{****} \cong X$.

Today: A new & conceptual proof of the above, using higher categories.

"related to $\pi_1(SO(3)) = \mathbb{Z}/2$"

"Inspired by the cobordism hypothesis"

<u>Thm</u> $\mathrm{Fun}_{\otimes}(\mathrm{Bord}^{or}_{1+1}, \mathcal{C}) \cong$ groupoid of commutative Frobenius algebras in $\mathcal{C}$.