A possible MO formulation of the \vee question

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Let $T=S^1\times S^1$ be an embedded 2-dimensional torus in S^4, and let γ_1 and γ_2 be $S^1\times\{1\}$ and $\{1\}\times S^1$ - both can be regarded as curves in S^4. Let Σ_1 and Σ_2 be two dimensional surface also embedded in S^4 and disjoint from T, such that the linking number of Σ_i with γ_i is 1, for $i=1,2$ (note that in S^4 there is a well-defined notion of a linking number between a curve and a surface disjoint from it, similar to the standard linking number between two disjoint curves in S^3).

Question. Do Σ_1 and Σ_2 necessarily intersect?

Example. Thinking of S^4 as a compactified ${\mathbb R}^4={\mathbb R}^2\times{\mathbb R}^2$, let T be the unit circle in the first copy of ${\mathbb R}^2$ times the unit circle in the second copy of ${\mathbb R}^2$, let $\Sigma_1=\{0\}\times{\mathbb R}^2$ and let $\Sigma_2 ={\mathbb R}^2\times\{0\}$. All conditions are satisfied and Σ_1 and Σ_2 intersect twice, once at 0 and once at ∞.

Let $T=S1\times S1$ be an embedded 2-dimensional torus in $S4$, and let $\gamma 1$ and $\gamma 2$ be $S1\times\{1\}$ and $\{1\}\times S1$ - both can be regarded as curves in $S4$. Let $\Sigma 1$ and $\Sigma 2$ be two dimensional surface also embedded in $S4$ and disjoint from T, such that the linking number of Σi with γi is 1, for $i=1,2$ (note that in $S4$ there is a well-defined notion of a linking number between a curve and a surface disjoint from it, similar to the standard linking number between two disjoint curves in $S3$).

Question. Do $\Sigma 1$ and $\Sigma 2$ necessarily intersect?

Example. Thinking of $S4$ as a compactified $\mathbb{R}4=\mathbb{R}2\times\mathbb{R}2$, let T be the unit circle in the first copy of $\mathbb{R}2$ times the unit circle in the second copy of $\mathbb{R}2$, let $\Sigma 1=\{0\}\times\mathbb{R}2$ and let $\Sigma 2=\mathbb{R}2\times\{0\}$. All conditions are satisfied and $\Sigma 1$ and $\Sigma 2$ intersect twice, once at 0 and once at ∞.

From <http://mathoverflow.net/questions/ask>