



$$K = X[3, 1, 4, 6] X[1, 5, 2, 4] X[5, 3, 6, 2]$$

$$X[1, 5, 2, 4] X[3, 1, 4, 6] X[5, 3, 6, 2]$$

$$t1 = K /. X[i_, j_, k_, l_] \to AP[i, j] P[k, l] + BP[i, l] P[j, k]$$

$$(BP[1, 4] P[3, 6] + AP[3, 1] P[4, 6])$$

$$(AP[1, 5] P[2, 4] + BP[1, 4] P[5, 2]) (BP[3, 6] P[5, 2] + AP[5, 3] P[6, 2])$$

$$t2 = \text{Expand}[t1]$$

$$\begin{aligned} & AB^2 P[1, 4] P[1, 5] P[2, 4] P[3, 6]^2 P[5, 2] + \\ & A^2 BP[1, 5] P[2, 4] P[3, 1] P[3, 6] P[4, 6] P[5, 2] + \\ & B^3 P[1, 4]^2 P[3, 6]^2 P[5, 2]^2 + AB^2 P[1, 4] P[3, 1] P[3, 6] P[4, 6] P[5, 2]^2 + \\ & A^2 BP[1, 4] P[1, 5] P[2, 4] P[3, 6] P[5, 3] P[6, 2] + \\ & A^3 P[1, 5] P[2, 4] P[3, 1] P[4, 6] P[5, 3] P[6, 2] + \\ & AB^2 P[1, 4]^2 P[3, 6] P[5, 2] P[5, 3] P[6, 2] + \\ & A^2 BP[1, 4] P[3, 1] P[4, 6] P[5, 2] P[5, 3] P[6, 2] \end{aligned}$$

$$\text{SetAttributes}[P, \text{Orderless}]$$

$$t2$$

$$\begin{aligned} & A^2 BP[1, 4] P[1, 5] P[2, 4] P[2, 6] P[3, 5] P[3, 6] + \\ & AB^2 P[1, 4]^2 P[2, 5] P[2, 6] P[3, 5] P[3, 6] + AB^2 P[1, 4] P[1, 5] P[2, 4] P[2, 5] P[3, 6]^2 + \\ & B^3 P[1, 4]^2 P[2, 5]^2 P[3, 6]^2 + A^3 P[1, 3] P[1, 5] P[2, 4] P[2, 6] P[3, 5] P[4, 6] + \\ & A^2 BP[1, 3] P[1, 4] P[2, 5] P[2, 6] P[3, 5] P[4, 6] + \\ & A^2 BP[1, 3] P[1, 5] P[2, 4] P[2, 5] P[3, 6] P[4, 6] + \\ & AB^2 P[1, 3] P[1, 4] P[2, 5]^2 P[3, 6] P[4, 6] \end{aligned}$$

t3 = t2 // . P[a_, b_] P[b_, c_] => P[a, c]

$$B^3 P[1, 4]^2 P[2, 5]^2 P[3, 6]^2 + AB^2 P[3, 6]^2 P[4, 5]^2 + \\ AB^2 P[2, 5]^2 P[4, 6]^2 + A^3 P[3, 5]^2 P[4, 6]^2 + 3A^2 B P[5, 6]^2 + AB^2 P[1, 4]^2 P[5, 6]^2$$

t4 = t3 /. P[i_, j_] ^ 2 => d

$$3A^2 B d + A^3 d^2 + 3AB^2 d^2 + B^3 d^3$$

t5 = Simplify[t4 /. {B -> 1/A, d -> (-A^2 - 1/A^2)}]

$$-\frac{1}{A^9} + \frac{1}{A} + A^3 + A^7$$