A Partial Reduction of BF Theory to Combinatorics

Abstract. I will describe a semi-rigorous reduction to computable combinatorics of perturbative BF theory (Cattaneo-Rossi [CR]), in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. Weak this result is, and at least partially already known (Watanabe [Wa]). Yet in the ribbon case, the resulting invariant is a universal finite type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Kashiwara-Vergne problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be as interesting.

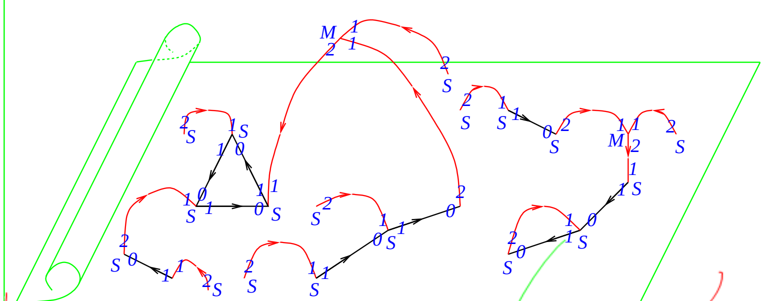
BF Following [CR]. $A \in \Omega^1(M = \mathbb{R}^4, \mathfrak{g})$, $B \in \Omega^2(\mathbb{R}^4, \mathfrak{g}^*)$,

$$S(A, B) := \int_{\mathbb{R}^4} \langle B, F_A \rangle.$$

With $f\colon (S = \mathbb{R}^2) \to \mathbb{R}^4$, $\xi \in \Omega^0(\mathbb{R}^2, \mathfrak{g})$, $\beta \in \Omega^1(\mathbb{R}^2, \mathfrak{g}^*)$, set

$$\mathcal{O}(A, B, f) := \int \mathcal{D}\xi \mathcal{D}\beta \exp\left(\frac{i}{\hbar} \int_{\mathbb{R}^2} \left\langle \xi, d_{f^*A}\beta + f^*B \right\rangle\right).$$

A BF Feynman Diagram.



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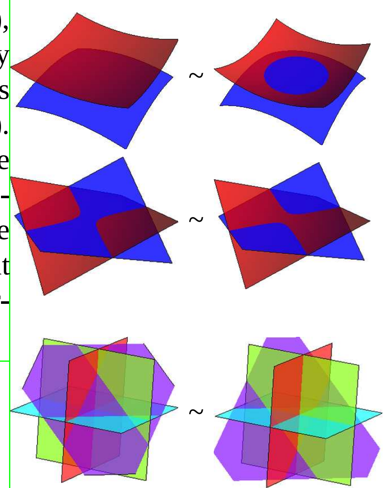
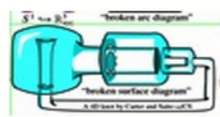


To do:

* BF box
* std. perturbation theory box.
* A word about axial gauge.
* decker set def box (3 generating figures, the xigg)
* decker set examples:

— 

— some example with triple points

* decker set moves.

References.

[CR] A. S. Cattaneo and C. A. Rossi, *Wilson Surfaces and Higher Dimensional Knot Invariants*, Commun. in Math. Phys. **256-3** (2005) 513–537, arXiv:math-ph/0210037.

[Wa] T. Watanabe, *Configuration Space Integrals for Long n-Knots, the Alexander Polynomial and Knot Space Cohomology*, Alg. and Geom. Top. **7** (2007) 47-92, arXiv:math/0609742.