The Growth Map

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continues "2013-12/I'm still missing a name for that".

$$e^{s\,\mathrm{ad}_u(\gamma)} = C_u^{\beta(s)}$$

$\ell \qquad s$

Find $\beta(s) := \Gamma_u(\gamma)$, "The growth map" on γ.

$\frac{d}{ds}$:

$$\mathrm{ad}_u\gamma /\!/ e^{s\,\mathrm{ad}_u(\gamma)} = \mathrm{ad}_u\left(\beta' /\!/ \frac{e^{\mathrm{ad}\,\beta(s)}-1}{\mathrm{ad}\,\beta(s)} /\!/ RC_u^{-\beta(s)}\right) /\!/ C_u^{\beta(s)}$$

$$\beta' /\!/ \frac{e^{\mathrm{ad}\,\beta(s)}-1}{\mathrm{ad}\,\beta(s)} /\!/ RC_u^{-\beta(s)} = \gamma$$

$$\beta' = \gamma /\!/ C_u^{\beta(s)} /\!/ \frac{\mathrm{ad}\,\beta(s)}{e^{\mathrm{ad}\,\beta(s)}-1}$$

$$= \gamma /\!/ e^{s\,\mathrm{ad}_u\gamma} /\!/ \frac{\mathrm{ad}\,\beta(s)}{e^{\mathrm{ad}\,\beta(s)}-1}$$

$\Rightarrow$ diffiQ

The many variable version.

$$\delta C_{u,v,\dots}^{\alpha,\beta,\dots} = \mathrm{ad}_{u,v\dots}\left\{\delta\alpha /\!/ \frac{e^{\mathrm{ad}\,\alpha}-1}{\mathrm{ad}\,\alpha} /\!/ RC_{u,v,\dots}^{-\alpha,-\beta,\dots},\ \delta\beta /\!/ \frac{e^{\mathrm{ad}\,\beta}-1}{\mathrm{ad}\,\beta} /\!/ RC_{u,v,\dots}^{-\alpha,-\beta,\dots}, \dots\right\} /\!/ C_{u,v,\dots}^{\alpha,\beta,\dots}$$

$$e^{s(\mathrm{ad}_u\alpha + \mathrm{ad}_v\beta)} = C_{u,v}^{\Gamma_{u,\alpha,\beta}(s),\ \Gamma_{v,\alpha,\beta}(s)}$$

unknowns are highlighted.

$\frac{d}{ds}$:

$$\mathrm{ad}_u\alpha + \mathrm{ad}_v\beta /\!/ e^{s(\dots)} = \mathrm{ad}_u\left(\Gamma_{u\alpha\beta}' /\!/ \frac{e^{\mathrm{ad}\,\Gamma_{u\alpha\beta}(s)}-1}{\mathrm{ad}\,\Gamma_{u\alpha\beta}(s)} /\!/ RC_{u,v}^{-\Gamma_{u\alpha\beta}(s),-\Gamma_{v\alpha\beta}(s)}\right) /\!/ C_{u,v}^{\Gamma_{u\alpha\beta}(s),\Gamma_{v\alpha\beta}(s)}$$

$$+ \mathrm{ad}_v\left(\Gamma_{v\alpha\beta}' /\!/ \frac{e^{\mathrm{ad}\,\Gamma_{v\alpha\beta}(s)}-1}{\mathrm{ad}\,\Gamma_{v\alpha\beta}(s)} /\!/ RC_{u,v}^{-\Gamma_{u\alpha\beta}(s),-\Gamma_{v\alpha\beta}(s)}\right) /\!/ C_{u,v}^{\Gamma_{u\alpha\beta}(s),\Gamma_{v\alpha\beta}(s)}$$

$\Rightarrow$

$$\alpha = \Gamma_{u\alpha\beta}' /\!/ \frac{e^{\mathrm{ad}\,\Gamma_{u\alpha\beta}(s)}-1}{\mathrm{ad}\,\Gamma_{u\alpha\beta}(s)} /\!/ RC_{u,v}^{-\Gamma_{u\alpha\beta}(s),-\Gamma_{v\alpha\beta}(s)}$$

$$\beta = \Gamma_{v\alpha\beta}' /\!/ \frac{e^{\mathrm{ad}\,\Gamma_{v\alpha\beta}(s)}-1}{\mathrm{ad}\,\Gamma_{v\alpha\beta}(s)} /\!/ RC_{u,v}^{-\Gamma_{u\alpha\beta}(s),-\Gamma_{v\alpha\beta}(s)}$$

$$\beta = \Gamma'_{v\alpha\beta} /\!/ \frac{e^{\cdots} - 1}{ad\,\Gamma_{v\alpha\beta}(s)} /\!/ RC_{u,v}^{-\Gamma_{u\alpha\beta}(s)), -\Gamma_{v\alpha\beta}(s)}$$

$$\Longrightarrow$$

$$\Gamma'_{u\alpha\beta} = \alpha /\!/ e^{s(ad_u \alpha + ad_v \beta)} /\!/ \frac{e^{ad\,\Gamma_{u\alpha\beta}(s)}}{e^{ad\,\Gamma_{u\alpha\beta}(s)} - 1}$$

$$\Gamma'_{v\alpha\beta} = \beta /\!/ e^{s(ad_u \alpha + ad_v \beta)} /\!/ \frac{e^{ad\,\Gamma_{v\alpha\beta}(s)}}{e^{ad\,\Gamma_{v\alpha\beta}(s)} - 1}$$