

Dror Bar-Natan: Talks: HUJI-140101: $\omega := \text{http://www.math.toronto.edu/~drorbn/Talks/HUJI-140101}$
 Handout and links at ω

Knots in 4 Dimensions and the Simplest Open Problem About Them

Abstract. I will describe a few 2-dimensional knots in 4 dimensional space in detail, then tell you how to make many more, then tell you that I don't really understand my way of making them, yet I can tell at least some of them apart in a colourful way.

Knots. "thermographical diagram" "broken arc diagram" in \mathbb{R}_{UV}^3

2-Knots. A 4D knot by Carter and Saito ω/CS

3-Colourings. Colour the arcs of a broken arc diagram in RGB so that every crossing is either mono-chromatic or tri-chromatic; $\lambda(K) := |\{3\text{-colourings}\}|$.
 Example. $\lambda(\bigcirc) = 3$ while $\lambda(\bigoplus) = 9$; so $\bigcirc \neq \bigoplus$.
 Exercise. Show that the set of colourings of K is a vector space over F_3 hence $\lambda(K)$ is always a power of 3.

Reidemeister's Theorem. Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemeister moves".
 $R1 = \text{twist}$, $R2 = \text{slide}$, $R3 = \text{crossing}$

Extend λ to wK by declaring that arcs "don't see" v-xings, and that caps are always "kosher". Then $\lambda(\bullet \rightarrow \bullet) = 3 \neq 9 = \lambda(\text{CS 2-knot})$, so assuming Conjecture, the CS 2-knot is indeed knotted.

The Generators
 "the crossing" ω/X
 "v-xing" ω/vX
 "cap" δ

The Double Inflation Procedure δ .

$wK := \text{PA} \langle \text{crossings} \rangle$
 $R2 = \text{slide}$, $R3 = \text{crossing}$, $M = \text{move}$, $VR1 = \text{v-reid}$, $VR2 = \text{v-reid}$, $VR3 = \text{v-reid}$, $OC = \text{open crossing}$, $CP = \text{cap}$, $UC = \text{uncap}$

"Planar Algebra": The objects are "tiles" that can be composed in arbitrary planar ways to make bigger tiles, which can then be composed even further....
 OC: as yet not UC:

Roseman Moves. ω/CS

Handwritten notes:
 rephrase as "2k = PAK \neq ..."
 proof by genericity / "shaking".
 rephrase as "2k = SA \neq ..."

Leopold Kronecker (modified): "God created the knots, all else in topology is the work of mortals."
 www.katlas.org