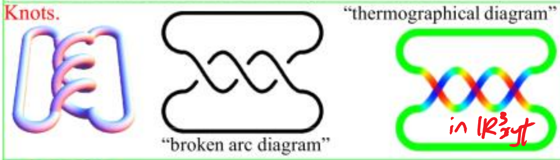
Eshnav Talk on 131216

December-13-13 9:06 AM

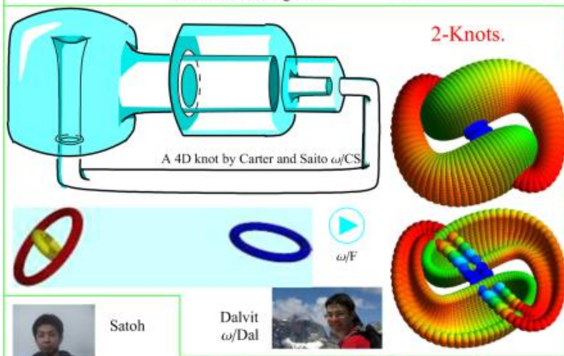
Knots in 4 Dimensions and the Simplest Open Problem About Them

Abstract. I will describe a few 2-dimensional knots in 4 dimensional space in detail, then tell you how to make many more, then tell you that I don't really understand my way of making them, yet I can tell at least some of them apart in a colourful way.

Knots.

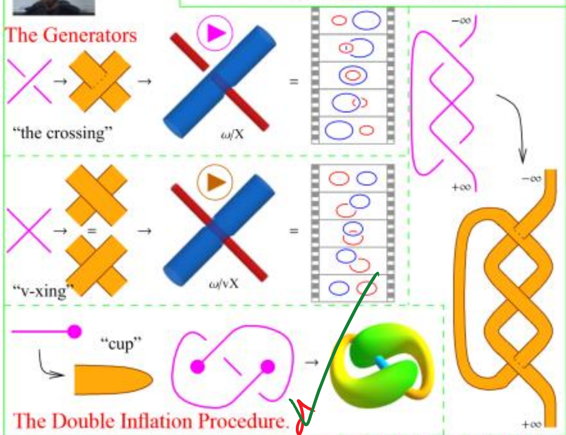


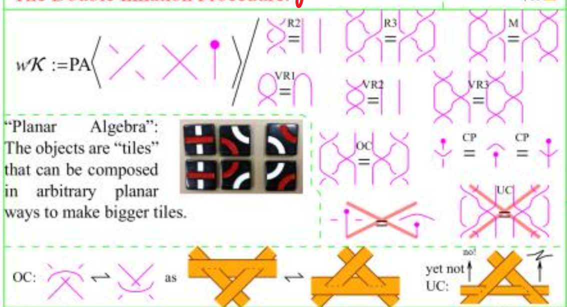
2-Knots.



The Generators

The Double Inflation Procedure.





"Planar Algebra": The objects are "tiles" that can be composed in arbitrary planar ways to make bigger tiles.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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Satoh's Conjecture. (ω/Sat) The "kernel" of the "double inflation" map δ, mapping "long" w-knot diagrams in the plane to "long" knotted 2D tubes in 4D, is precisely the moves R1–R3, VR1–VR3, D and OC listed below.

rephrase

What's "the same"

In other words, two long w-knot diagrams represent via δ the same long 2D knotted tube in 4D iff they differ by a sequence of the said moves.

First Iso. Thm: $\delta : G \to H \Rightarrow \operatorname{im}\delta \cong G/\ker(\delta)$

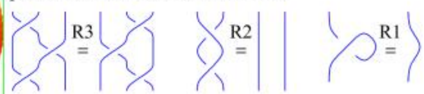
δ is a map from algebra to topology. So a thing in "hard" topology ("~~ribbon 2-knots~~" im δ) is the same as a thing in "easy" algebra (wK).

3-Colourings. Colour the arcs of a broken arc diagram in RGB so that every crossing is either mono-chromatic or tri-chromatic; $\lambda(K) := |\{3\text{-colourings}\}|$.

Example. $\lambda(\bigcirc) = 3$ while $\lambda(\text{trefoil}) = 9$; so $\bigcirc \neq$ trefoil.

Exercise: $\lambda(K)$ is always a power of 3.

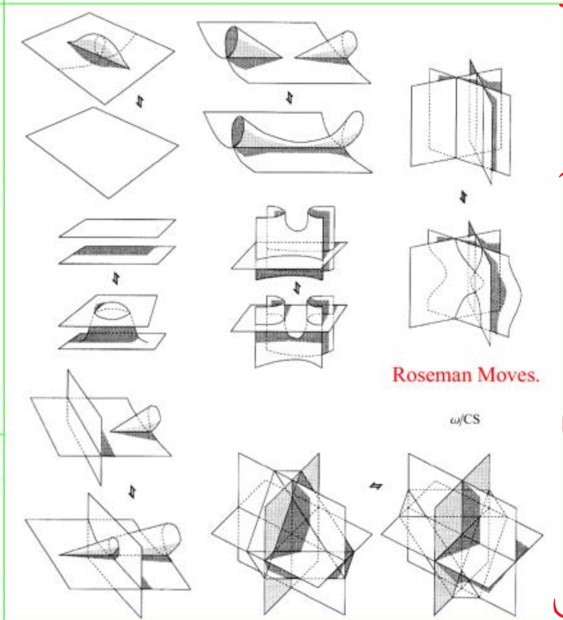
Reidemeister' Theorem. Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemeister moves":



Kurt Reidemeister

shrink

The wK extension. Conditional thms



shrink much.

add some 3-colourings nonsense.