Eshnav Talk Post-Mortem

January-02-14 7:26 AM

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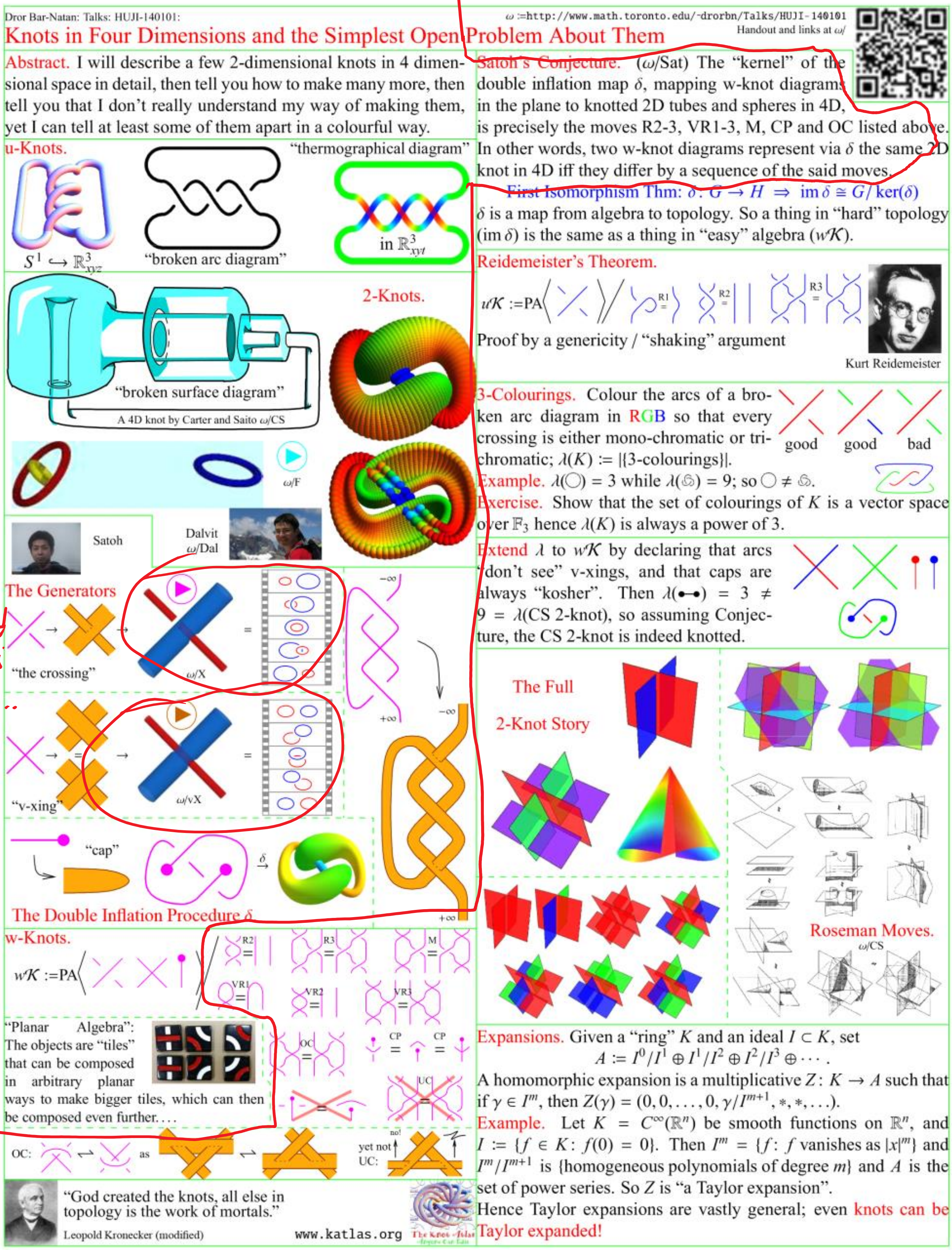

Dror Bar-Natan: Talks: HUJI-140101: ω :=http://www.math.toronto.edu/~drorbn/Talks/HUJI-140101 Handout and links at ω/

Knots in Four Dimensions and the Simplest Open Problem About Them

Abstract. I will describe a few 2-dimensional knots in 4 dimensional space in detail, then tell you how to make many more, then tell you that I don't really understand my way of making them, yet I can tell at least some of them apart in a colourful way.

u-Knots. "thermographical diagram" $S^1 \hookrightarrow \mathbb{R}^3_{xyz}$ "broken arc diagram" in $\mathbb{R}^3_{xyt}$

"broken surface diagram" A 4D knot by Carter and Saito ω/CS

2-Knots. ω/F

Satoh Dalvit ω/Dal

The Generators "the crossing" ω/X "v-xing" ω/vX $-\infty$ $+\infty$ "cap"

The Double Inflation Procedure δ

w-Knots. $w\mathcal{K} :=$ PA$\langle \ldots \rangle$ R2 R3 M VR1 VR2 VR3 OC CP CP UC

"Planar Algebra": The objects are "tiles" that can be composed in arbitrary planar ways to make bigger tiles, which can then be composed even further....

OC: as yet not UC: no!

"God created the knots, all else in topology is the work of mortals." Leopold Kronecker (modified) www.katlas.org

Satoh's Conjecture. (ω/Sat) The "kernel" of the double inflation map δ, mapping w-knot diagrams in the plane to knotted 2D tubes and spheres in 4D, is precisely the moves R2-3, VR1-3, M, CP and OC listed above. In other words, two w-knot diagrams represent via δ the same 2D knot in 4D iff they differ by a sequence of the said moves.

First Isomorphism Thm: $\delta: G \to H \Rightarrow \operatorname{im}\delta \cong G/\ker(\delta)$

δ is a map from algebra to topology. So a thing in "hard" topology ($\operatorname{im}\delta$) is the same as a thing in "easy" algebra ($w\mathcal{K}$).

Reidemeister's Theorem. $u\mathcal{K} :=$ PA$\langle \ldots \rangle$ R1 R2 R3

Proof by a genericity / "shaking" argument

Kurt Reidemeister

3-Colourings. Colour the arcs of a broken arc diagram in RGB so that every crossing is either mono-chromatic or tri-chromatic; $\lambda(K) := |\{3\text{-colourings}\}|$. good good bad

Example. $\lambda(\bigcirc) = 3$ while $\lambda(\text{trefoil}) = 9$; so $\bigcirc \neq$ trefoil.

Exercise. Show that the set of colourings of K is a vector space over $\mathbb{F}_3$ hence $\lambda(K)$ is always a power of 3.

Extend λ to $w\mathcal{K}$ by declaring that arcs "don't see" v-xings, and that caps are always "kosher". Then $\lambda(\bullet\!\!-\!\!\bullet) = 3 \neq 9 = \lambda(\text{CS 2-knot})$, so assuming Conjecture, the CS 2-knot is indeed knotted.

The Full 2-Knot Story

Roseman Moves. ω/CS

Expansions. Given a "ring" K and an ideal $I \subset K$, set
$$A := I^0/I^1 \oplus I^1/I^2 \oplus I^2/I^3 \oplus \cdots.$$
A homomorphic expansion is a multiplicative $Z\colon K \to A$ such that if $\gamma \in I^m$, then $Z(\gamma) = (0, 0, \ldots, 0, \gamma/I^{m+1}, *, *, \ldots)$.

Example. Let $K = C^\infty(\mathbb{R}^n)$ be smooth functions on $\mathbb{R}^n$, and $I := \{f \in K\colon f(0) = 0\}$. Then $I^m = \{f\colon f \text{ vanishes as } |x|^m\}$ and I^m/I^{m+1} is {homogeneous polynomials of degree m} and A is the set of power series. So Z is "a Taylor expansion".

Hence Taylor expansions are vastly general; even knots can be Taylor expanded!

I should have indicated which axis is what...

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1. I could/should have avoided the discussion of planar algebras.