

Crossed Modules

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From Kauffman - Faria-Martins:

2.3 Crossed module invariants of knotted surfaces

A crossed module (see [Bro99]) $\mathcal{G} = (E \xrightarrow{\partial} G, \triangleright)$ is given by a group morphism $\partial: E \rightarrow G$ together with a left action \triangleright of G on E by automorphisms. The conditions on ∂ and \triangleright are:

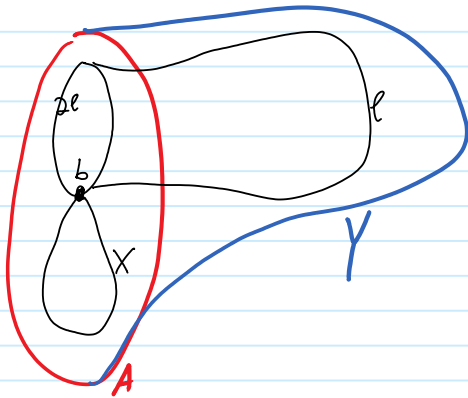
- (i) $\partial(X \triangleright e) = X \partial(e) X^{-1}$, for all $X \in G$, for all $e \in E$; ✓
- (ii) $\partial(e) \triangleright f = e f e^{-1}$, for all $e, f \in E$.

Example: $A \subset Y$ topological spaces,

$$G = \pi_1(A) \quad E = \pi_2(Y, A) = \left[\begin{array}{c} b \\ \square \\ Y \\ A \end{array} \right]$$

∂ - the standard connecting homomorphism.

\triangleright - the standard action of π_1 on π_2 by "pulling the lip".



Alternatively,

