

From 2013-07/CUMC-2013 Post Mortem:

I should have put a "references" section with refs to some good knot theory books.

Dror Bar-Natan: Talks: ~~CUMC 2007~~
 $\omega := \text{http://www.math.toronto.edu/~drorbn/Talks/CUMC-2007}$

Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.

Visualizing the Fourth Dimension

4D Knots.
 A 4D knot by Carter and Saito ω/CS



with ~~EM Dalvit~~
 ω/Dal



Flatlanders View an Elephant.



"The third dimension isn't t"

ω/g ω/r ω/b coords from $\omega/Jeff2207$

A Simplified Notation / Double Inflation



ω/X
 ω/vX
 ω/F
 $\omega/X1$

Knots.





ω/P
 $\omega/M1$
 $\omega/M2$

The Double Inflation Procedure δ .




$\delta:$ ω/inf
 $\delta:$ "long w-knot diagram" \rightarrow "long knotted 2D tube in 4D"





Banks like knots. Which knot appears twice?

Many of the images are by Carter and Carter-Saito, ω/CS .



Carter, Banach, Saito

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Dror Bar-Natan: Talks: CUMC-1307:
 $\omega := \text{http://www.math.toronto.edu/~drorbn/Talks/CUMC-1307}$

and the Simplest Thing I Don't Know About It

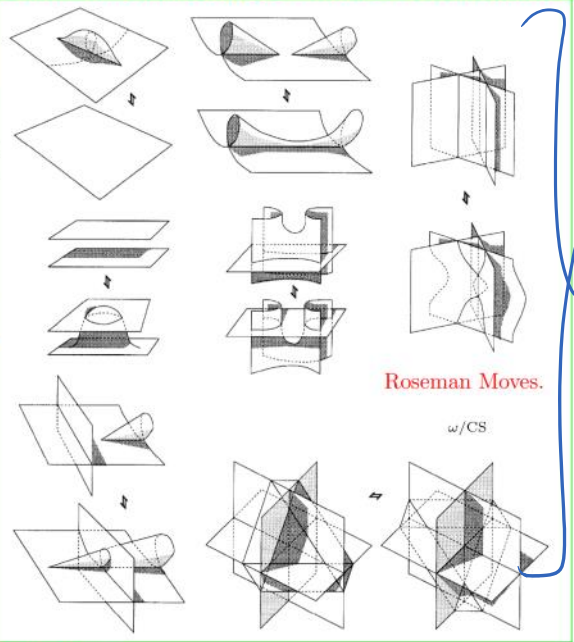
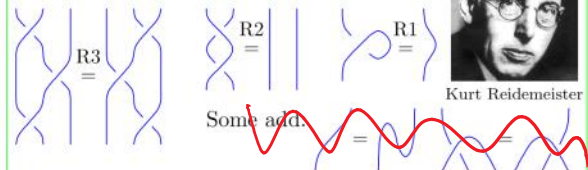
Satoh's Conjecture. (ω/Sat) The "kernel" of the "double inflation" map δ , mapping "long" w-knot diagrams in the plane to "long" knotted 2D tubes in 4D, is precisely the moves R1-R3, VR1-VR3, D and OC listed below.



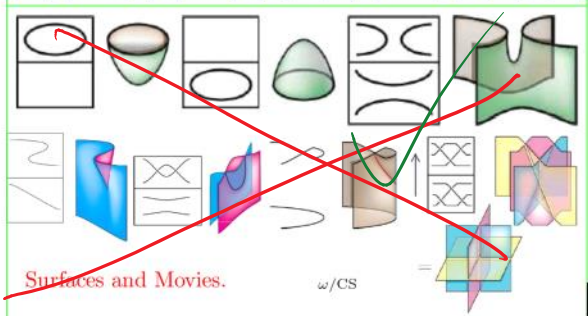
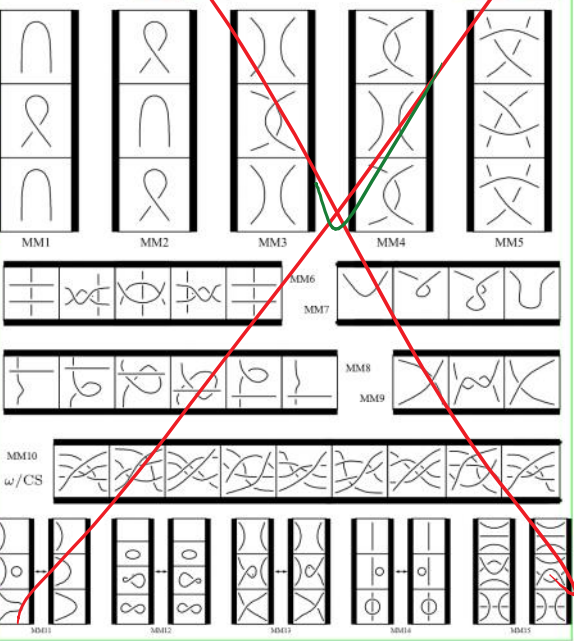
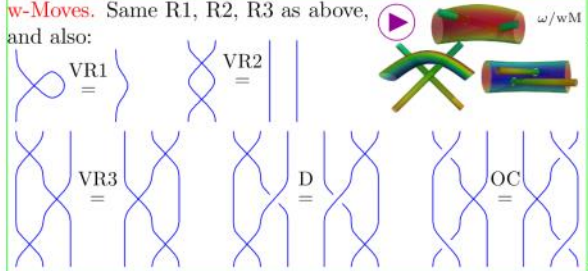
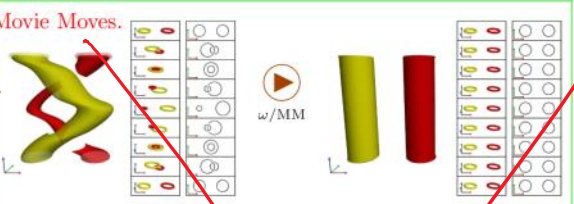
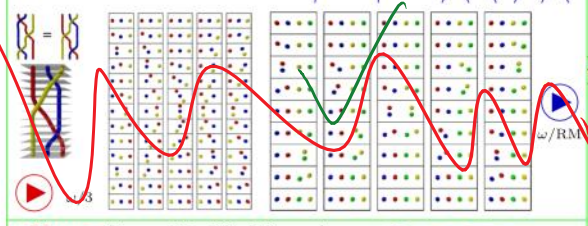
In other words, two long w-knot diagrams represent via δ the same long 2D knotted tube in 4D iff they differ by a sequence of the said moves.

First Iso Thm: $\phi: G \rightarrow H \Rightarrow \text{im } \phi \cong G / \ker(\phi)$
 δ is a map from algebra to topology. So a thing in "hard" topology ("ribbon 2-knots") is the same as a thing in "easy" algebra. **What's "The Same"?**

Reidemeister' Theorem. Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemeister moves":



Can do my own!



"God created the knots, all else in topology is the work of mortals."
 Leopold Kronecker (modified) www.katlas.org The Knot Atlas

