

Dror Bar-Natan: Talks: ClassroomAdventures-1401:  
 $\omega := \text{http://www.math.toronto.edu/~drorbn/Talks/ClassroomAdventures-1401}$

### Visualizing the Fourth Dimension

#### 4D Knots.

A 4D knot by Carter and Saito  $\omega/CS$

with Ester Dalvit  $\omega/Dal$

*add the "other" CS knot.*

**Abstract.** Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.

#### Flatlanders View an Elephant.

"The third dimension isn't *t*"

$\omega/g$   $\omega/r$   $\omega/b$  coords from  $\omega/Jeff2207$

#### A Simplified Notation / Double Inflation

$\omega/X$   $\omega/vX$   $\omega/F$   $\omega/X1$

#### Knots.

$\omega/M2$

#### The Double Inflation Procedure $\delta$ .

$\delta$ :  $\omega/inf$

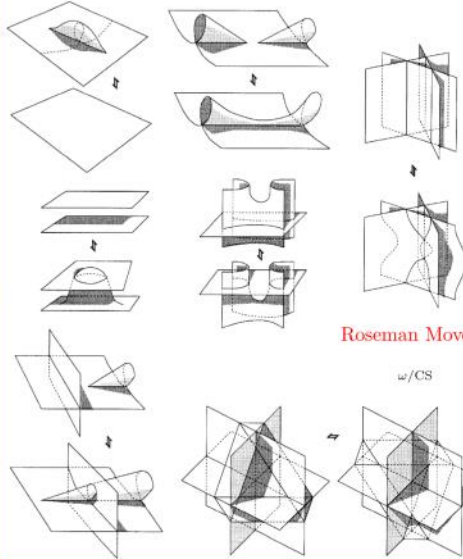
$\delta$ : "long w-knot diagram"  $\rightarrow$  "long knotted 2D tube in 4D"

#### Banks like knots. Which knot appears twice?

Many of the images are by Carter and Carter-Saito,  $\omega/CS$ .

Carter, Banach, Saito

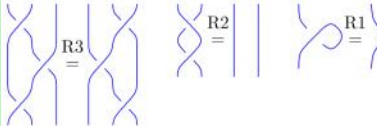
**Satoh's Conjecture.** ( $\omega$ /Sat) Two long w-knot diagrams represent via the double inflation map  $\delta$  the same long 2D knotted tube in 4D iff they differ by a sequence of the said moves R1-R3, VR1-VR3, D and OC listed below.



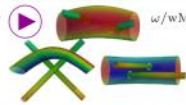
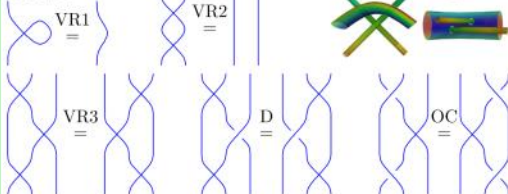
Can do my own!

What's "The Same"?

**Reidemeister' Theorem.** Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemeister moves":



**w-Moves.** Same R1, R2, R3 as above, and also:



add the double inflation in age.

Some knot theory references.



"God created the knots, all else in topology is the work of mortals."  
 Leopold Kronecker (modified)

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