
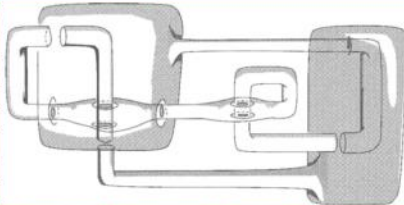


\* still too many popups! [though they can harmlessly be skipped]

Dror Bar-Natan: Talks: ClassroomAdventures-1401  
 $\omega := \text{http://www.math.toronto.edu/~drorbn/Talks/ClassroomAdventures-1401}$

**Abstract.** Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.

**Visualizing the Fourth Dimension** 

  $\omega/CS$   
 Yet another 4D Knot.

**A Simplified Notation / Double Inflation**



**Flatlanders View an Elephant.**

 "The third dimension isn't t"  
 $\omega/g$   $\omega/r$   $\omega/b$  coords from  $\omega/Jeff2207$

**Knots.**

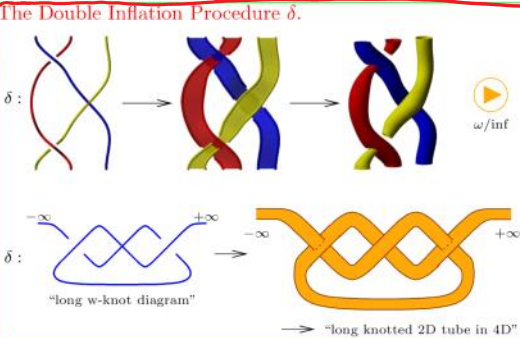
 with Ester Dalvit  $\omega/Dal$   $\omega/M2$

**4D Knots.**

A 4D knot by Carter and Saito  $\omega/CS$

 Carter, Banach, Saito

**Satoh's Conjecture.** (Satoh, *Virtual Knot Presentations of Ribbon Torus-Knots*, J. Knot Theory and its Ramifications **9** (2000) 531–542). Two long w-knot diagrams represent via the double inflation map  $\delta$  the same long 2D knotted tube in 4D iff they differ by a sequence of the "w-moves" R1–R3, VR1–VR3, D and OC listed below.

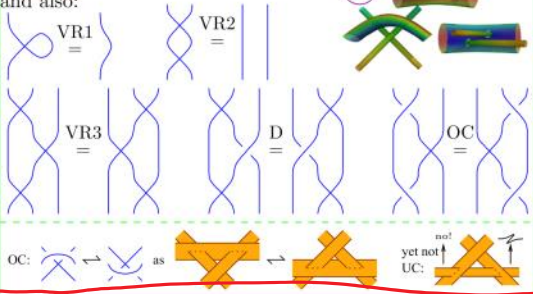
  $\delta$ :  $\omega/inf$   
 $\delta$ : "long w-knot diagram"  $\rightarrow$  "long knotted 2D tube in 4D"

**Reidemeister' Theorem.** Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemeister moves":

 R3 = R2 = R1 =  Kurt Reidemeister

*Handwritten note:* detail line rush line

w-Moves. Same R1, R2, R3 as above, and also:

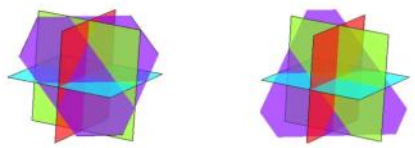
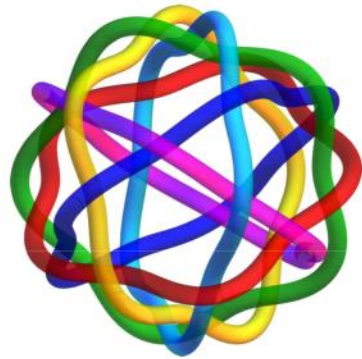
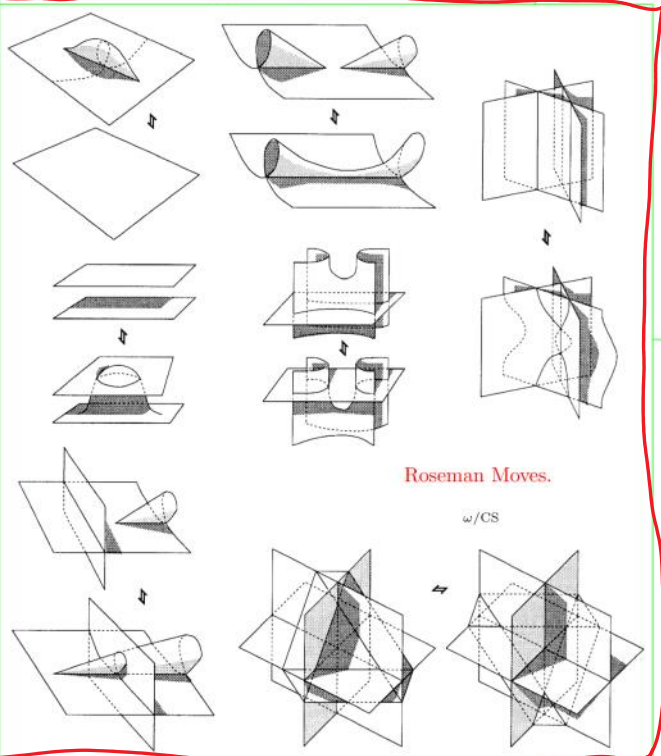


Some knot theory books.

Colin C. Adams, *The Knot Book, an Elementary Introduction to the Mathematical Theory of Knots*, American Mathematical Society, 2004.  
 Meike Akveld and Andrew Jobbins, *Knots Unravelled, from Strings to Mathematics*, Arbelos 2011.  
 J. Scott Carter and Masahico Saito, *Knotted Surfaces and Their Diagrams*, American Mathematical Society, 1997.  
 Peter Cromwell, *Knots and Links*, Cambridge University Press, 2004.  
 W.B. Raymond Lickorish, *An Introduction to Knot Theory*, Springer 1997.



not done



"God created the knots, all else in topology is the work of mortals."  
 Leopold Kronecker (modified) [www.katlas.org](http://www.katlas.org)

Banks like knots. Which knot appears twice?