

Pensieve header: Computation with Huan Vo regarding Stavros' "Planar Limit".

```
<< KnotTheory`
```

```
Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.
```

```
Read more at http://katlas.org/wiki/KnotTheory.
```

```
PlanarLimit[knot_][a_] := HOMFLYPT[knot][a, 0]
```

```
(* the planar limit of a knot *)
```

```
PL[P_, Q_, a_] := PlanarLimit[Mirror[TorusKnot[P, Q]]][a]
```

```
HOMFLYPT[PD[Loop[1], Loop[2]]][a, z]
```

```
KnotTheory::credits : The HOMFLYPT program was written by Scott Morrison.
```

$$-\frac{1}{az} + \frac{a}{z}$$

```
HOMFLYPT[Link[2, Alternating, 1]][a, z]
```

```
KnotTheory::loading : Loading precomputed data in PD4Links`.
```

$$-\frac{a}{z} + \frac{a^3}{z} - az$$

```
HOMFLYPT[#][a, z] & /@ AllKnots[{3, 5}]
```

```
KnotTheory::loading : Loading precomputed data in PD4Knots`.
```

$$\left\{ 2a^2 - a^4 + a^2z^2, -1 + \frac{1}{a^2} + a^2 - z^2, 3a^4 - 2a^6 + 4a^4z^2 - a^6z^2 + a^4z^4, a^2 + a^4 - a^6 + a^2z^2 + a^4z^2 \right\}$$

```
PlanarLimit[#][a] & /@ AllKnots[{3, 10}] // Union // Length
```

```
142
```

```
Conway[#][a] & /@ AllKnots[{3, 10}] // Union // Length
```

```
211
```

```
D[Conway[#][a], a, a, a] & /@ AllKnots[{3, 10}] // Union // Length
```

```
67
```

**D[Conway[#][a], a, a, a] & /@ AllKnots[{3, 10}]**

```
{0, 0, 24 a, 0, 0, -24 a, 24 a, 120 a + 120 a^3, 0, 48 a, 0, 48 a, -24 a, 24 a, 0, -72 a - 120 a^3,
0, -48 a, -72 a - 120 a^3, -48 a, 72 a + 120 a^3, 48 a, -72 a - 120 a^3, 72 a + 120 a^3, -48 a,
24 a, 48 a, -48 a, 72 a, 48 a + 120 a^3, -48 a - 120 a^3, -24 a - 120 a^3, 120 a + 120 a^3,
24 a, -24 a, 360 a + 840 a^3 + 336 a^5, 0, 216 a + 240 a^3, 72 a, 0, 192 a + 240 a^3, 72 a,
-48 a, 192 a + 240 a^3, 96 a, -24 a - 120 a^3, -48 a, 96 a, 48 a, -48 a, 168 a + 240 a^3,
24 a + 120 a^3, 96 a, 48 a, -24 a - 120 a^3, -48 a, 24 a + 120 a^3, 96 a, -24 a - 120 a^3,
-72 a, 24 a + 120 a^3, -24 a - 120 a^3, 24 a + 120 a^3, 24 a + 120 a^3, -24 a - 120 a^3,
24 a + 120 a^3, 120 a^3, -120 a^3, -120 a^3, 0, -24 a - 120 a^3, 48 a, 120 a, -72 a,
-24 a + 120 a^3, 72 a, -24 a, -72 a - 120 a^3, 24 a, -24 a, 0, 48 a + 120 a^3, -24 a, 72 a,
0, -120 a - 600 a^3 - 336 a^5, 0, -72 a, 168 a + 600 a^3 + 336 a^5, -144 a - 240 a^3, -72 a,
-168 a - 240 a^3, -168 a - 600 a^3 - 336 a^5, 72 a, -96 a, 144 a + 240 a^3, 48 a, -96 a - 240 a^3,
144 a + 240 a^3, -96 a, 168 a + 600 a^3 + 336 a^5, -96 a, 120 a + 240 a^3, -72 a, -120 a - 240 a^3,
-144 a - 240 a^3, 120 a + 240 a^3, -96 a, -96 a - 240 a^3, -120 a - 240 a^3, 96 a + 240 a^3,
96 a, -24 a + 120 a^3, -96 a, 96 a, -96 a - 240 a^3, 96 a, 72 a, 48 a, -72 a, 96 a, -96 a,
-96 a - 240 a^3, 96 a + 240 a^3, -24 a + 120 a^3, 24 a - 120 a^3, 24 a - 120 a^3, -24 a + 120 a^3,
24 a - 120 a^3, -144 a - 600 a^3 - 336 a^5, 192 a + 600 a^3 + 336 a^5, 192 a + 600 a^3 + 336 a^5,
240 a + 360 a^3, -120 a - 240 a^3, 120 a + 240 a^3, 120 a + 240 a^3, 144 a, 144 a + 240 a^3,
120 a, -96 a - 240 a^3, 96 a + 240 a^3, 72 a, -24 a + 120 a^3, 24 a - 120 a^3, -168 a - 240 a^3,
192 a + 600 a^3 + 336 a^5, 120 a, -192 a - 600 a^3 - 336 a^5, 120 a + 240 a^3, 216 a + 360 a^3,
-96 a, 96 a, -24 a + 120 a^3, -24 a + 120 a^3, 24 a - 120 a^3, -72 a - 240 a^3, -24 a + 120 a^3,
-96 a, 24 a - 120 a^3, -120 a - 240 a^3, 120 a + 240 a^3, 24 a - 120 a^3, 216 a + 600 a^3 + 336 a^5,
216 a + 360 a^3, 48 a - 120 a^3, -96 a - 480 a^3 - 336 a^5, 72 a + 240 a^3, 72 a + 240 a^3,
96 a + 480 a^3 + 336 a^5, -72 a - 240 a^3, -72 a - 240 a^3, 48 a - 120 a^3, -48 a + 120 a^3,
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72 a + 240 a^3, 24 a - 120 a^3, -120 a, -72 a - 240 a^3, 144 a + 480 a^3 + 336 a^5,
120 a + 480 a^3 + 336 a^5, 168 a, -96 a - 240 a^3, 96 a + 240 a^3, 120 a + 480 a^3 + 336 a^5,
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72 a - 120 a^3, -48 a - 360 a^3 - 336 a^5, 48 a + 240 a^3, 48 a + 360 a^3 + 336 a^5, -48 a - 240 a^3,
192 a, 24 a + 240 a^3, -24 a - 240 a^3, -24 a + 240 a^3 + 336 a^5, 336 a + 840 a^3 + 336 a^5,
96 a + 120 a^3, 96 a + 120 a^3, -48 a - 120 a^3, 216 a + 240 a^3, 48 a, 48 a, -48 a, 24 a,
-24 a, 192 a + 240 a^3, 72 a, -24 a, 24 a, 24 a + 120 a^3, 336 a + 840 a^3 + 336 a^5,
24 a, -72 a - 120 a^3, 216 a + 240 a^3, 72 a + 120 a^3, -72 a, 24 a, 48 a, -48 a,
72 a + 120 a^3, -24 a - 120 a^3, -48 a - 120 a^3, 48 a + 120 a^3, 312 a + 840 a^3 + 336 a^5,
120 a + 120 a^3, 144 a + 120 a^3, -72 a - 120 a^3, 48 a + 120 a^3, -120 a^3, -48 a - 120 a^3,
48 a + 120 a^3, -48 a - 120 a^3, 144 a + 120 a^3, -72 a, 24 a + 120 a^3, 72 a, -48 a}
```

**CE = GatherBy[AllKnots[{3, 10}], Conway[#][z] &]**

```
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**CE** /. **K\_Knot**  $\Rightarrow$  **PlanarLimit[K][a]**

**KnotTheory::credits** : The HOMFLYPT program was written by Scott Morrison.

$$\left\{ \{2 a^2 - a^4\}, \left\{-1 + \frac{1}{a^2} + a^2\right\}, \{3 a^4 - 2 a^6, 3 a^4 - 2 a^6\}, \{a^2 + a^4 - a^6\}, \right. \\
\left. \left\{\frac{1}{a^2} - a^2 + a^4, 2 - a^2 - a^4 + a^6\right\}, \{2 - 2 a^2 + a^4\}, \left\{3 - \frac{1}{a^2} - a^2\right\}, \{4 a^6 - 3 a^8\}, \{a^2 + a^6 - a^8\}, \right. \\
\left. \left\{-\frac{2}{a^8} + \frac{2}{a^6} + \frac{1}{a^4}\right\}, \left\{-\frac{1}{a^8} + \frac{2}{a^4}, a^2 + a^8 - a^{10}\right\}, \{2 a^4 - a^8, -1 + 2 a^2 + 2 a^4 - 2 a^6\}, \right.$$

$$\begin{aligned}
& \left\{ 1 - a^2 + 2 a^4 - a^6, a^2 + 2 a^4 - 3 a^6 + a^8 \right\}, \left\{ 2 + \frac{1}{a^4} - \frac{2}{a^2} \right\}, \left\{ \frac{1}{a^2} - a^4 + a^6 \right\}, \left\{ 3 a^2 - 3 a^4 + a^6 \right\}, \\
& \left\{ -1 + \frac{1}{a^4} + a^4, \frac{1}{a^2} - a^6 + a^8 \right\}, \left\{ -2 + \frac{2}{a^2} + a^4 \right\}, \left\{ \frac{2}{a^6} - \frac{5}{a^4} + \frac{4}{a^2}, 2 - 2 a^2 + a^4 \right\}, \left\{ 2 - a^2 - a^4 + a^6 \right\}, \\
& \left\{ -1 - \frac{2}{a^4} + \frac{4}{a^2} \right\}, \left\{ 2 - \frac{1}{a^4} + \frac{1}{a^2} - a^2, 2 - \frac{1}{a^2} + a^2 - a^4 \right\}, \left\{ -3 + \frac{2}{a^2} + 2 a^2, 3 + \frac{2}{a^4} - \frac{4}{a^2} \right\}, \\
& \left\{ -2 - \frac{3}{a^4} + \frac{6}{a^2}, 3 a^4 - 2 a^6 \right\}, \left\{ 1 + a^2 - 2 a^4 + a^6, -1 + \frac{1}{a^2} + a^2 \right\}, \left\{ 1 + \frac{1}{a^4} - \frac{1}{a^2} - a^2 + a^4 \right\}, \\
& \left\{ -\frac{1}{a^4} + \frac{2}{a^2} \right\}, \left\{ 1, -1 + \frac{1}{a^2} + 2 a^4 - a^6, 2 a^2 - 2 a^6 + a^8 \right\}, \left\{ a^4 + 3 a^6 - 4 a^8 + a^{10} \right\}, \\
& \left\{ 2 a^2 - a^4, 2 a^2 - a^4 \right\}, \left\{ -1 + \frac{1}{a^2} + a^2 \right\}, \left\{ 3 - \frac{1}{a^2} - a^2, -3 + \frac{1}{a^2} + 5 a^2 - 2 a^4 \right\}, \left\{ \frac{1}{a^{10}} - \frac{5}{a^8} + \frac{5}{a^6} \right\}, \\
& \left\{ -1 + 4 a^2 - 2 a^4, 1 - 2 a^2 + 4 a^4 - 2 a^6 \right\}, \left\{ 3 a^2 - 3 a^4 + a^6, -2 - \frac{1}{a^4} + \frac{3}{a^2} + a^2 \right\}, \\
& \left\{ 5 a^8 - 4 a^{10} \right\}, \left\{ -\frac{3}{a^{10}} + \frac{3}{a^8} + \frac{1}{a^6} \right\}, \left\{ a^4 + 2 a^8 - 2 a^{10} \right\}, \left\{ -\frac{1}{a^{10}} + \frac{1}{a^6} + \frac{1}{a^4} \right\}, \\
& \left\{ 3 a^6 - a^8 - a^{10} \right\}, \left\{ 2 a^4 - a^6 + a^8 - a^{10} \right\}, \left\{ 2 a^6 + a^8 - 2 a^{10} \right\}, \left\{ -\frac{2}{a^{10}} + \frac{1}{a^8} + \frac{2}{a^6} \right\}, \\
& \left\{ -\frac{2}{a^8} + \frac{3}{a^6} - \frac{1}{a^4} + \frac{1}{a^2} \right\}, \left\{ 1 - a^4 + 2 a^6 - a^8 \right\}, \left\{ -\frac{1}{a^{10}} - \frac{1}{a^8} + \frac{3}{a^6} \right\}, \left\{ 1 + \frac{1}{a^6} - \frac{2}{a^4} + \frac{1}{a^2} \right\}, \\
& \left\{ 1 - \frac{1}{a^8} + \frac{1}{a^6} + \frac{1}{a^4} - \frac{1}{a^2}, -\frac{1}{a^6} + \frac{1}{a^4} + \frac{1}{a^2} \right\}, \left\{ -\frac{3}{a^8} + \frac{4}{a^6} \right\}, \left\{ -3 + \frac{2}{a^2} + 2 a^2 \right\}, \left\{ a^4 + a^6 - a^{10} \right\}, \\
& \left\{ \frac{1}{a^4} - \frac{1}{a^2} + a^2 \right\}, \left\{ 2 a^2 - 2 a^4 + 2 a^6 - a^8, 4 a^4 - 4 a^6 + a^8 \right\}, \left\{ -\frac{1}{a^8} + \frac{1}{a^6} + \frac{1}{a^2} \right\}, \\
& \left\{ -4 - \frac{1}{a^4} + \frac{4}{a^2} + 2 a^2 \right\}, \left\{ a^4 + 2 a^6 - 2 a^8 \right\}, \left\{ 1 + a^2 - 3 a^4 + 3 a^6 - a^8 \right\}, \left\{ \frac{1}{a^6} - \frac{3}{a^4} + \frac{3}{a^2} \right\}, \\
& \left\{ -2 + \frac{1}{a^2} + 3 a^2 - a^4 \right\}, \left\{ -1 + 5 a^2 - 4 a^4 + a^6, -3 + \frac{1}{a^2} + 5 a^2 - 2 a^4, 1 - \frac{1}{a^6} + \frac{2}{a^4} - \frac{1}{a^2} \right\}, \\
& \left\{ -4 + \frac{2}{a^2} + 4 a^2 - a^4 \right\}, \left\{ -1 + 4 a^2 - 2 a^4 \right\}, \left\{ 1 + \frac{1}{a^6} - \frac{2}{a^4} + \frac{1}{a^2} \right\}, \left\{ 2 a^2 - a^4 \right\}, \left\{ -1 + \frac{1}{a^2} + a^2 \right\}, \\
& \left\{ 3 a^6 - a^8 - a^{10} \right\}, \left\{ -\frac{2}{a^8} + \frac{4}{a^6} - \frac{3}{a^4} + \frac{2}{a^2} \right\}, \left\{ -2 + \frac{1}{a^4} + 2 a^2 \right\}, \left\{ 4 a^6 - 3 a^8, a^4 + 3 a^8 - 4 a^{10} + a^{12} \right\}, \\
& \left\{ -\frac{1}{a^8} + \frac{2}{a^6} - \frac{2}{a^4} + \frac{2}{a^2} \right\}, \left\{ 2 - 2 a^2 + a^4, -2 + \frac{1}{a^6} - \frac{3}{a^4} + \frac{4}{a^2} + a^2 \right\}, \left\{ 3 a^2 - 3 a^4 + a^6 \right\}, \\
& \left\{ -3 + \frac{2}{a^2} + 2 a^2 \right\}, \left\{ -\frac{1}{a^8} + \frac{3}{a^6} - \frac{4}{a^4} + \frac{3}{a^2} \right\}, \left\{ -2 + \frac{1}{a^2} + 3 a^2 - a^4 \right\}, \left\{ 2 a^2 - 2 a^4 + 2 a^6 - a^8 \right\}, \\
& \left\{ 1 + \frac{1}{a^6} - \frac{2}{a^4} + \frac{1}{a^2} \right\}, \left\{ -\frac{2}{a^6} + \frac{3}{a^4} \right\}, \left\{ -\frac{3}{a^8} + \frac{4}{a^6} \right\}, \left\{ 4 a^4 - 4 a^6 + a^8 \right\}, \left\{ \frac{1}{a^4} - a^2 + a^6 \right\}, \\
& \left\{ \frac{2}{a^4} - \frac{2}{a^2} + a^4 \right\}, \left\{ -\frac{3}{a^6} + \frac{5}{a^4} - \frac{1}{a^2} \right\}, \left\{ 3 a^2 - 2 a^4 - a^6 + a^8 \right\}, \left\{ 1 + a^4 - 2 a^6 + a^8 \right\}, \\
& \left\{ 3 - 3 a^2 + a^6 \right\}, \left\{ 3 + \frac{2}{a^4} - \frac{4}{a^2} \right\}, \left\{ 1 - \frac{1}{a^6} + \frac{2}{a^4} - \frac{1}{a^2}, 3 - \frac{1}{a^2} - a^2 \right\}, \left\{ -1 + \frac{2}{a^2} - a^2 + a^6 \right\}, \\
& \left\{ -1 - \frac{2}{a^6} + \frac{2}{a^4} + \frac{2}{a^2}, 3 - \frac{2}{a^4} + \frac{2}{a^2} - 2 a^2 \right\}, \left\{ -1 + \frac{1}{a^4} + a^2 - a^4 + a^6 \right\}, \left\{ a^2 + a^4 - a^6 \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{1 - \frac{2}{a^4} + \frac{3}{a^2} - a^2\right\}, \left\{1 + \frac{1}{a^6} - \frac{2}{a^2} + a^2\right\}, \left\{5 - \frac{2}{a^2} - 2a^2\right\}, \left\{\frac{1}{a^2} - a^2 + a^4, 1 + a^2 - a^4 - a^6 + a^8\right\}, \\
& \left\{3 - \frac{1}{a^2} - a^2\right\}, \left\{2 - a^2 - a^6 + a^8, 3 - 3a^2 + a^6\right\}, \left\{a^2 + 2a^4 - 3a^6 + a^8\right\}, \left\{-1 + \frac{2}{a^4} - \frac{2}{a^2} + 2a^2\right\}, \\
& \left\{-\frac{2}{a^6} + \frac{3}{a^4}, 4 - \frac{1}{a^4} - 2a^2\right\}, \left\{2a^2 - 2a^6 + a^8, \frac{1}{a^8} - \frac{2}{a^6} + \frac{2}{a^2}\right\}, \left\{1 + \frac{2}{a^4} - \frac{3}{a^2} + a^2\right\}, \\
& \left\{a^2 + a^4 - a^6\right\}, \left\{-1 - \frac{1}{a^6} + \frac{3}{a^2}, 1 - \frac{1}{a^4} + \frac{1}{a^2} + a^2 - a^4\right\}, \left\{-2 + \frac{2}{a^2} + a^2 - a^4 + a^6\right\}, \left\{2a^2 - a^4\right\}, \\
& \left\{2 - \frac{1}{a^4} + \frac{1}{a^2} - a^2, a^2 + a^4 - a^6\right\}, \left\{-1 + \frac{1}{a^2} + a^2\right\}, \{1\}, \left\{2 - \frac{1}{a^6} + \frac{1}{a^4} - a^2, 4 - \frac{2}{a^2} - a^4\right\}, \\
& \left\{1 + \frac{1}{a^6} - \frac{1}{a^4} - a^2 + a^4\right\}, \left\{1 - a^2 + 2a^4 - a^6\right\}, \left\{1 + a^2 - 2a^4 + a^6\right\}, \left\{2a^2 - a^4\right\}, \\
& \left\{-1 - \frac{1}{a^6} + \frac{3}{a^2}, -1 + 3a^2 - a^6\right\}, \left\{-1 + \frac{1}{a^2} + 2a^2 - 2a^4 + a^6\right\}, \left\{-2 - \frac{1}{a^4} + \frac{3}{a^2} + a^2, \frac{1}{a^6} - \frac{3}{a^4} + \frac{3}{a^2}\right\}, \\
& \left\{-1 - \frac{1}{a^4} + \frac{2}{a^2} + 2a^2 - a^4\right\}, \left\{-2 + \frac{1}{a^2} + 3a^2 - a^4\right\}, \left\{-3 + \frac{2}{a^2} + 2a^2\right\}, \left\{\frac{3}{a^8} - \frac{8}{a^6} + \frac{6}{a^4}\right\}, \\
& \left\{-\frac{5}{a^6} + \frac{9}{a^4} - \frac{3}{a^2}\right\}, \left\{9 - \frac{4}{a^2} - 4a^2\right\}, \left\{a^6 + 5a^8 - 7a^{10} + 2a^{12}\right\}, \left\{\frac{2}{a^8} - \frac{4}{a^6} + \frac{1}{a^4} + \frac{2}{a^2}\right\}, \\
& \left\{-1 - \frac{3}{a^6} + \frac{4}{a^4} + \frac{1}{a^2}\right\}, \left\{3a^6 - 3a^{10} + a^{12}\right\}, \left\{a^4 + a^6 + a^8 - 3a^{10} + a^{12}\right\}, \left\{-1 - \frac{2}{a^6} + \frac{2}{a^4} + \frac{2}{a^2}\right\}, \\
& \left\{-2 + \frac{1}{a^4} + 3a^2 - 2a^4 + a^6\right\}, \left\{-2 + \frac{1}{a^2} + 4a^2 - 3a^4 + a^6\right\}, \left\{4 + \frac{1}{a^6} + \frac{1}{a^4} - \frac{5}{a^2}\right\}, \left\{-\frac{4}{a^6} + \frac{7}{a^4} - \frac{2}{a^2}\right\}, \\
& \left\{4 + \frac{3}{a^4} - \frac{6}{a^2}\right\}, \left\{-\frac{3}{a^6} + \frac{5}{a^4} - \frac{1}{a^2}, -2 - \frac{1}{a^6} - \frac{1}{a^4} + \frac{5}{a^2}\right\}, \left\{2a^6 + 2a^8 - 4a^{10} + a^{12}\right\}, \\
& \{1, 2a^2 - 2a^6 + a^8\}, \left\{-\frac{1}{a^8} + \frac{2}{a^6} - \frac{2}{a^4} + \frac{2}{a^2}\right\}, \left\{-3 + \frac{1}{a^6} - \frac{2}{a^4} + \frac{3}{a^2} + 2a^2\right\}, \\
& \left\{-3 - \frac{1}{a^4} + \frac{3}{a^2} + 3a^2 - a^4\right\}, \left\{-\frac{1}{a^8} + \frac{2}{a^6} - \frac{2}{a^4} + \frac{2}{a^2}\right\}, \left\{3a^2 - 4a^4 + 3a^6 - a^8\right\}, \\
& \left\{\frac{1}{a^8} - \frac{4}{a^4} + \frac{4}{a^2}\right\}, \left\{a^2 - a^4 + 4a^6 - 4a^8 + a^{10}\right\}, \left\{11 - \frac{5}{a^2} - 5a^2\right\}, \left\{2a^6 + 3a^8 - 6a^{10} + 2a^{12}\right\}, \\
& \left\{1 - \frac{1}{a^4} + \frac{1}{a^2} + a^2 - a^4\right\}, \{1\}, \left\{1 - \frac{1}{a^6} + \frac{2}{a^4} - \frac{1}{a^2}\right\}, \left\{-1 - \frac{2}{a^4} + \frac{4}{a^2}\right\}, \left\{a^2 + a^4 - a^6\right\}, \\
& \left\{2 + \frac{1}{a^4} - \frac{2}{a^2}\right\}, \left\{-2 - \frac{1}{a^4} + \frac{3}{a^2} + a^2, a^2 + 3a^4 - 5a^6 + 2a^8\right\}, \left\{-1 + \frac{1}{a^2} + a^2\right\}, \left\{1 - a^4 + 2a^6 - a^8\right\}, \\
& \left\{-2 + \frac{1}{a^4} + 2a^2\right\}, \left\{5 - \frac{2}{a^2} - 2a^2\right\}, \left\{-\frac{1}{a^6} + \frac{1}{a^4} + \frac{1}{a^2}\right\}, \left\{2a^2 - a^4\right\}, \left\{3 + \frac{2}{a^4} - \frac{4}{a^2}\right\}, \left\{-\frac{2}{a^6} + \frac{3}{a^4}\right\}, \\
& \left\{-3 + \frac{1}{a^6} - \frac{2}{a^4} + \frac{3}{a^2} + 2a^2\right\}, \left\{-\frac{1}{a^8} + \frac{2}{a^6} - \frac{2}{a^4} + \frac{2}{a^2}\right\}, \left\{9 - \frac{4}{a^2} - 4a^2\right\}, \left\{-a^2 + 5a^4 - 3a^6\right\}, \\
& \left\{\frac{1}{a^{12}} - \frac{4}{a^{10}} + \frac{2}{a^8} + \frac{2}{a^6}\right\}, \left\{\frac{1}{a^4} - \frac{1}{a^2} + a^2\right\}, \left\{3 - \frac{1}{a^2} - a^2\right\}, \left\{-1 + \frac{1}{a^2} + a^2\right\}, \left\{2 + \frac{1}{a^4} - \frac{2}{a^2}\right\}, \\
& \left\{-\frac{1}{a^4} + \frac{2}{a^2}\right\}, \{1\}, \left\{7 - \frac{3}{a^2} - 3a^2\right\}, \left\{\frac{1}{a^2} - a^4 + a^6\right\}, \left\{\frac{1}{a^8} - \frac{3}{a^6} + \frac{2}{a^4} + \frac{1}{a^2}\right\}, \left\{-1 + 4a^2 - 2a^4\right\}, \\
& \left\{\frac{1}{a^6} - \frac{3}{a^4} + \frac{3}{a^2}\right\}, \left\{2a^2 - a^4\right\}, \left\{3 - \frac{1}{a^2} - a^2\right\}, \{1\}, \left\{-\frac{1}{a^6} + \frac{1}{a^4} + \frac{1}{a^2}\right\}, \{1\}, \left\{-1 + \frac{1}{a^2} + a^2\right\},
\end{aligned}$$

$$\begin{aligned}
& \{3 a^6 - 3 a^{10} + a^{12}\}, \{1 - a^2 + 2 a^4 - a^6\}, \left\{-1 - \frac{2}{a^4} + \frac{4}{a^2}\right\}, \left\{-3 + \frac{2}{a^2} + 2 a^2\right\}, \left\{\frac{2}{a^{12}} - \frac{8}{a^{10}} + \frac{7}{a^8}\right\}, \\
& \left\{7 - \frac{3}{a^2} - 3 a^2\right\}, \{-2 a^2 + 7 a^4 - 4 a^6\}, \{5 a^4 - 6 a^6 + 2 a^8, -\frac{1}{a^4} + \frac{2}{a^2}\}, \left\{\frac{1}{a^{12}} - \frac{4}{a^{10}} + \frac{2}{a^8} + \frac{2}{a^6}\right\}, \\
& \left\{\frac{1}{a^{12}} - \frac{3}{a^{10}} + \frac{3}{a^6}\right\}, \left\{-1 + \frac{1}{a^2} + 2 a^2 - 2 a^4 + a^6\right\}, \left\{-3 + \frac{1}{a^6} - \frac{2}{a^4} + \frac{3}{a^2} + 2 a^2\right\}, \left\{\frac{1}{a^{12}} - \frac{6}{a^{10}} + \frac{6}{a^8}\right\}, \\
& \left\{\frac{1}{a^{12}} - \frac{5}{a^{10}} + \frac{4}{a^8} + \frac{1}{a^6}\right\}, \{3 - 4 a^2 + 2 a^4\}, \{2 a^4 - a^6 + a^8 - a^{10}\}, \{1\}, \{-a^2 + 5 a^4 - 3 a^6\}, \\
& \left\{-1 - \frac{1}{a^6} + \frac{3}{a^2}\right\}, \{8 a^8 - 10 a^{10} + 3 a^{12}\}, \left\{6 - \frac{3}{a^2} - a^2 - a^4\right\}, \left\{\frac{1}{a^{12}} - \frac{2}{a^{10}} - \frac{2}{a^8} + \frac{4}{a^6}\right\}, \\
& \left\{-\frac{1}{a^8} + \frac{2}{a^4}\right\}, \left\{-2 + \frac{1}{a^4} + 2 a^2\right\}, \{a^2 + a^4 - a^6\}, \left\{-\frac{1}{a^8} + \frac{1}{a^6} + \frac{1}{a^2}\right\}, \{3 a^6 - a^8 - a^{10}\}
\end{aligned}$$