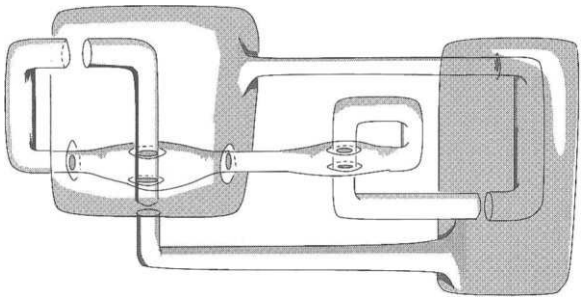
Dror Bar-Natan: Talks: ClassroomAdventures-1401:
ω :=http://www.math.toronto.edu/~drorbn/Talks/ClassroomAdventures-1401

Visualizing the Fourth Dimension

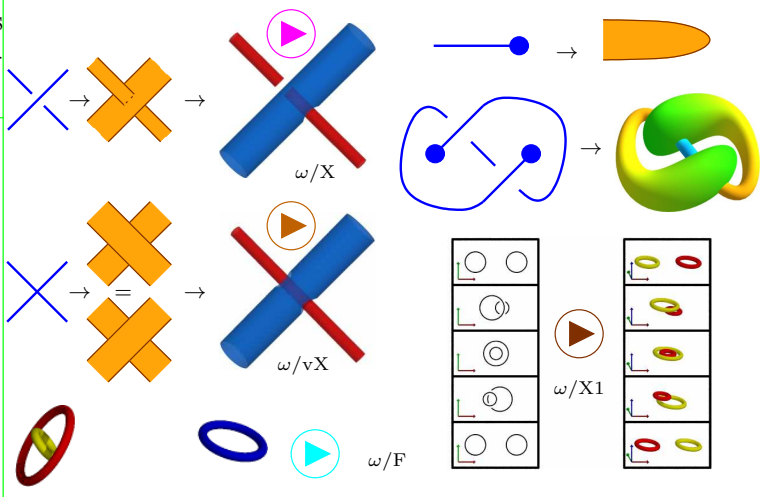
Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.



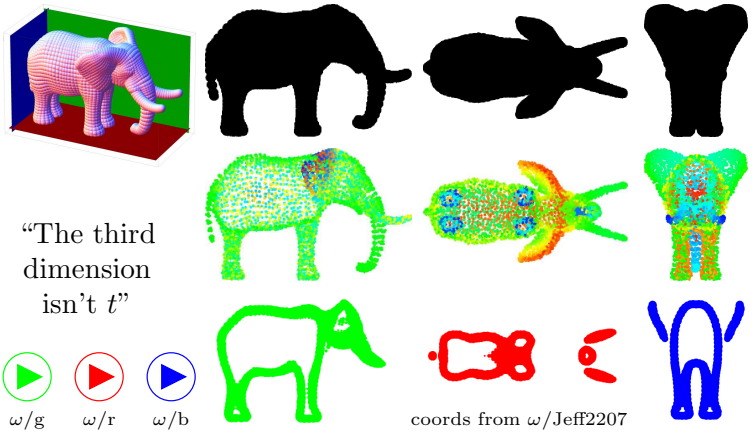
ω/CS

Yet another 4D Knot.

A Simplified Notation / Double Inflation



Flatlanders View an Elephant.

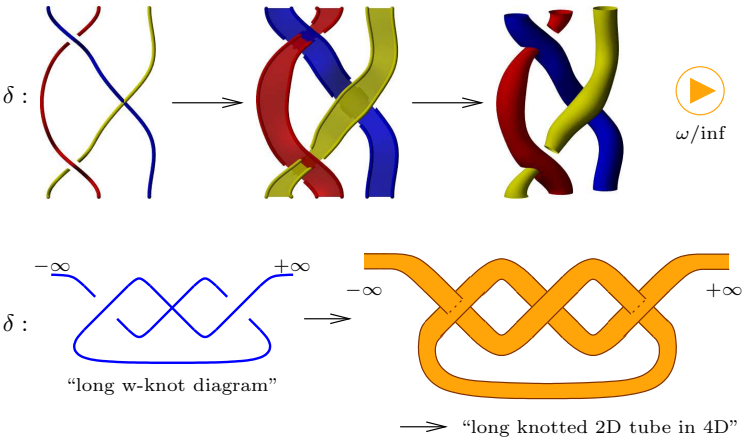


"The third dimension isn't t"

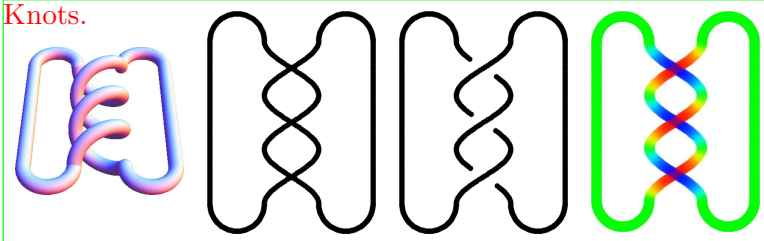
ω/g ω/r ω/b

coords from ω/Jeff2207

The Double Inflation Procedure δ.



Knots.



with Ester Dalvit

ω/Dal

ω/M2

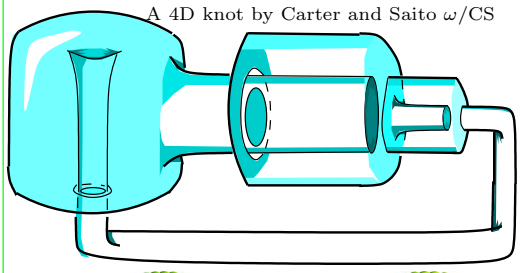
Satoh's Conjecture.

(Satoh, *Virtual Knot Presentations of Ribbon Torus-Knots,* J. Knot Theory and its Ramifications **9** (2000) 531–542). Two long w-knot diagrams represent via the double inflation map δ the same long 2D knotted tube in 4D iff they differ by a sequence of the "w-moves" R1–R3, VR1–VR3, D and OC listed below.

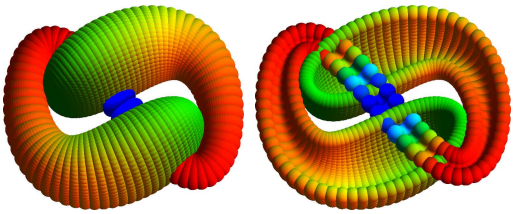
Shin Satoh

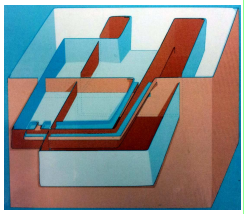
4D Knots.

A 4D knot by Carter and Saito ω/CS



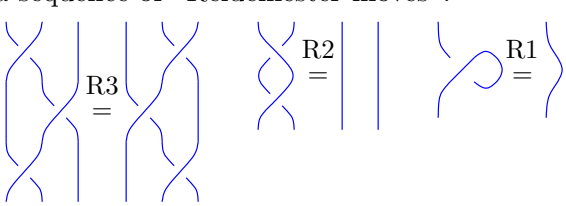
Carter, Banach, Saito





Reidemeister' Theorem.

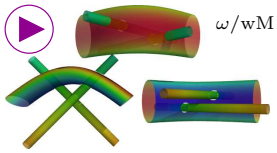
Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemester moves":



Kurt Reidemeister

and the Simplest Thing I Don't Know About It

w-Moves. Same R1, R2, R3 as above, and also:



Some knot theory books.

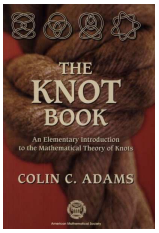
Colin C. Adams, *The Knot Book, an Elementary Introduction to the Mathematical Theory of Knots,* American Mathematical Society, 2004.

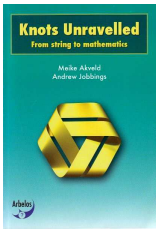
Meike Akveld and Andrew Jobbings, *Knots Unravelled, from Strings to Mathematics,* Arbelos 2011.

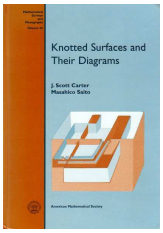
J. Scott Carter and Masahico Saito, *Knotted Surfaces and Their Diagrams,* American Mathematical Society, 1997.

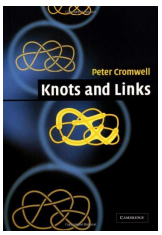
Peter Cromwell, *Knots and Links,* Cambridge University Press, 2004.

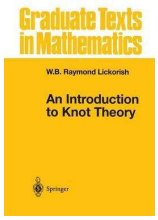
W.B. Raymond Lickorish, *An Introduction to Knot Theory,* Springer 1997.

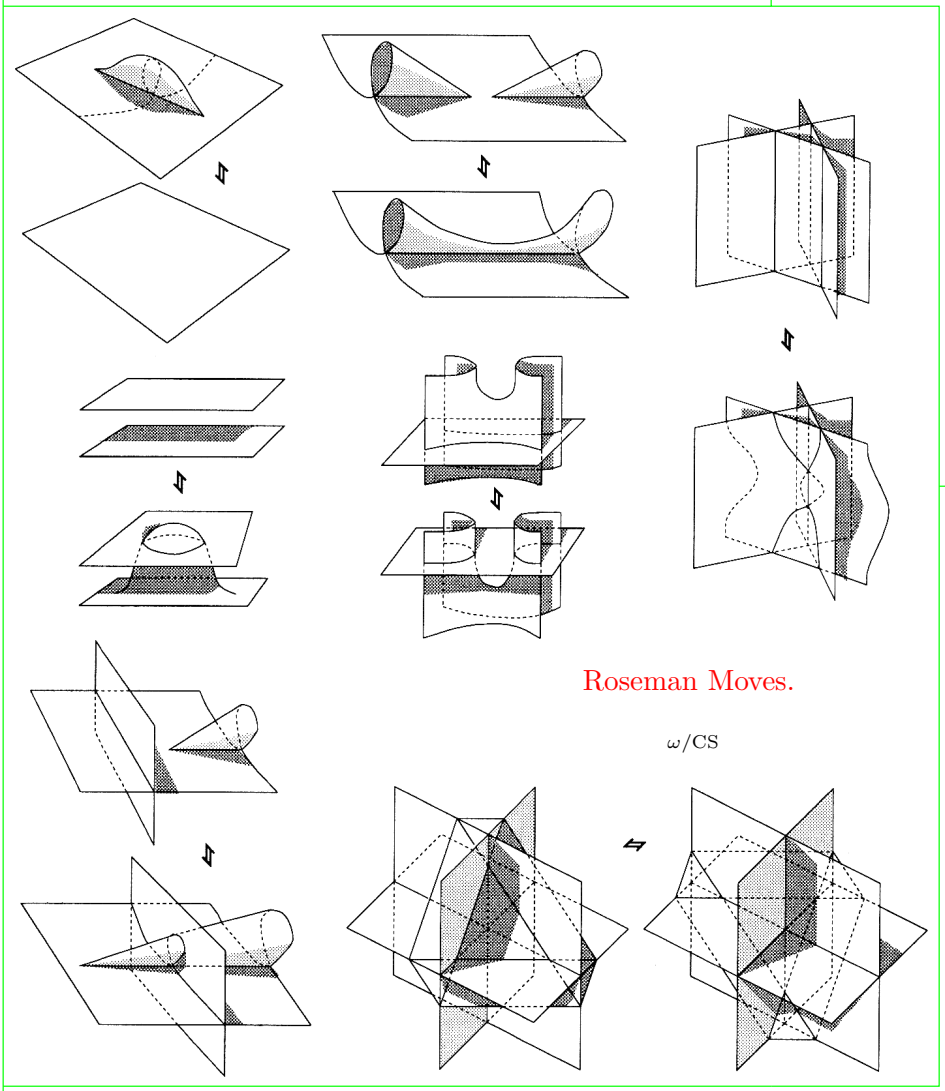




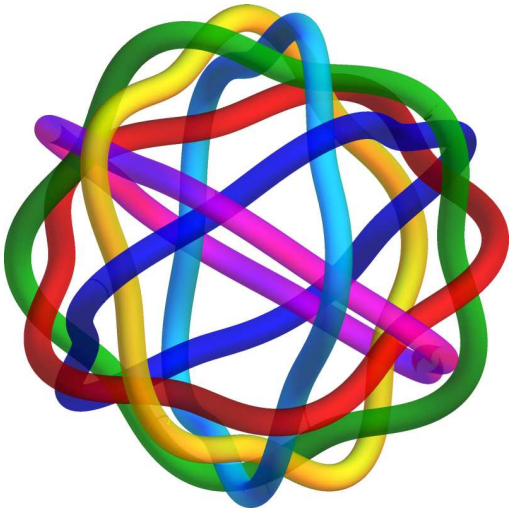


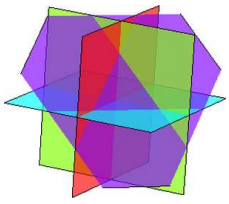


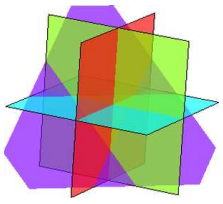


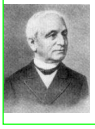


Roseman Moves.









"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Banks like knots. which knot appears twice?