Cheat Sheet J

http://drorbn.net/AcademicPensieve/2014-01/
initiated 18/3/13; continues 2013-12; continued CheatSheetFreeLie @ Projects/WKO4; modified 1/9/14, 10:11am

With alphabet T and with $u, v, w \in T$, $\alpha, \beta, \gamma \in FL(T)$, $D \in \mathfrak{tder}(T)$, $g, h \in \exp(\mathfrak{tder}(T)) = \mathrm{TAut}(T)$. Checkmarks (✓) as in `CheatSheetJ-Verification.nb`.

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1. The definition of J: $\quad J_u(\gamma) := \int_0^1 ds\, \mathrm{div}_u(\gamma \sslash RC_u^{s\gamma}) \sslash C_u^{-s\gamma}$

2. ✓ The J_{uv} equation: $\quad J_u(\alpha) + J_v(\beta \sslash RC_u^\alpha) \sslash C_u^{-\alpha} = J_v(\beta) + J_u(\alpha \sslash RC_v^\beta) \sslash C_v^{-\beta}$

3. ✓ The t equation: $\quad J_w(\gamma \sslash tm_w^{uv}) = (J_u(\gamma) + J_v(\gamma \sslash RC_u^\gamma) \sslash C_u^{-\gamma}) \sslash tm_w^{uv}$

4. ✓ The h equation: $\quad J_u(\mathrm{bch}(\alpha,\beta)) = J_u(\alpha) + J_u(\beta \sslash RC_u^\alpha) \sslash C_u^{-\alpha}$

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5. ✓ The meaning(s) of RC: $\quad C_u^\gamma \sslash RC_u^{-\gamma} = Id, \qquad C_u^{\gamma \sslash RC_u^\gamma} = RC_u^\gamma$

6. ✓ C_uC_v and RC_uRC_v: $\quad C_u^{\alpha \sslash RC_v^{-\beta}} \sslash C_v^\beta = C_v^{\beta \sslash RC_u^{-\alpha}} \sslash C_u^\alpha, \qquad RC_u^\alpha \sslash RC_v^{\beta \sslash RC_u^\alpha} = RC_v^\beta \sslash RC_u^{\alpha \sslash RC_v^\beta}$

7. RC equation t: $\quad tm_w^{uv} \sslash RC_w^{\gamma \sslash tm_w^{uv}} = RC_u^\gamma \sslash RC_v^{\gamma \sslash RC_u^\gamma} \sslash tm_w^{uv}$

8. RC equation h: $\quad RC_u^{\mathrm{bch}(\alpha,\beta)} = RC_u^\alpha \sslash RC_u^{\beta \sslash RC_u^\alpha}$

9. C–div–RC equations: $\quad \mathrm{div}_u(\alpha \sslash RC_u^\gamma) \sslash C_u^\gamma = ? \qquad \mathrm{div}_u(\alpha \sslash C_u^\gamma) \sslash RC_u^\gamma = ?$

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10. div property t: $\quad \mathrm{div}_w(\gamma \sslash tm_w^{uv}) = (\mathrm{div}_u(\gamma) + \mathrm{div}_v(\gamma)) \sslash tm_w^{uv}$

11. ✓ div property uv: with $\mathrm{ad}_u^\gamma = \mathrm{ad}_u\{\gamma\} := \mathrm{der}(u \to [\gamma, u])$,
$$(\mathrm{div}_u\,\alpha) \sslash \mathrm{ad}_v^\beta - (\mathrm{div}_v\,\beta) \sslash \mathrm{ad}_u^\alpha = \mathrm{div}_u(\alpha \sslash \mathrm{ad}_v^\beta) - \mathrm{div}_v(\beta \sslash \mathrm{ad}_u^\alpha)$$

12. ✓ div property uu: $\quad (\mathrm{div}_u\,\alpha) \sslash \mathrm{ad}_u\{\beta\} - (\mathrm{div}_u\,\beta) \sslash \mathrm{ad}_u\{\alpha\} = \mathrm{div}_u\left([\alpha,\beta] + \alpha \sslash \mathrm{ad}_u\{\beta\} - \beta \sslash \mathrm{ad}_u\{\alpha\}\right)$

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13. The definition of JA: $\quad JA_u(\gamma) := J_u(\gamma) \sslash RC_u^\gamma$

14. The ODE for JA: with $\gamma_s = \gamma \sslash RC_u^{s\gamma}$, $\quad JA(0) = 0, \quad \dfrac{dJA(s)}{ds} = JA(s) \sslash \mathrm{ad}_u\{\gamma_s\} + \mathrm{div}_u\,\gamma_s, \quad JA(1) = JA_u(\gamma)$

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15. ✓ The growth map $\Gamma_u(\gamma) := \beta(1)$. With $\beta(0) = 0$ and $\beta'(s) = \gamma \sslash e^{\mathrm{ad}_u\{s\gamma\}} \sslash \frac{\mathrm{ad}\,\beta(s)}{e^{\mathrm{ad}\,\beta(s)}-1}$, $\qquad e^{\mathrm{ad}_u\{\gamma\}} = C_u^{\beta(1)}$

16. ✓ Many-variable growth. With $\Gamma_{u\alpha\beta}(0) = \Gamma_{v\alpha\beta}(0) = 0$, $\Gamma'_{u\alpha\beta}(s) = \alpha \sslash e^{\mathrm{ad}_u\{s\alpha\}+\mathrm{ad}_v\{s\beta\}} \sslash \frac{\mathrm{ad}\,\Gamma_{u\alpha\beta}(s)}{e^{\mathrm{ad}\,\Gamma_{u\alpha\beta}(s)}-1}$ and $\Gamma'_{v\alpha\beta}(s) = \beta \sslash e^{\mathrm{ad}_u\{s\alpha\}+\mathrm{ad}_v\{s\beta\}} \sslash \frac{\mathrm{ad}\,\Gamma_{v\alpha\beta}(s)}{e^{\mathrm{ad}\,\Gamma_{v\alpha\beta}(s)}-1}$, $\qquad e^{\mathrm{ad}_u\{s\alpha\}+\mathrm{ad}_v\{s\beta\}} = C_{u,v}^{\Gamma_{u\alpha\beta}(s),\Gamma_{v\alpha\beta}(s)}$

17. The definition of j (following A-T): $\quad j(e^D) = \int_0^1 ds\, e^{sD}(\mathrm{div}\,D) = \dfrac{e^D - 1}{D}(\mathrm{div}\,D)$

18. j's cocycle property: $\quad j(gh) = j(g) + g \cdot j(h)$

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19. The differential of exp: $\quad \delta e^\gamma = e^\gamma \cdot \left(\delta\gamma \sslash \dfrac{1 - e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right) = \left(\delta\gamma \sslash \dfrac{e^{\mathrm{ad}\,\gamma} - 1}{\mathrm{ad}\,\gamma}\right) \cdot e^\gamma$

20. ✓ The differential of $\gamma = \mathrm{bch}(\alpha,\beta)$: $\quad \delta\gamma \sslash \dfrac{1 - e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} = \left(\delta\alpha \sslash \dfrac{1 - e^{-\mathrm{ad}\,\alpha}}{\mathrm{ad}\,\alpha} \sslash e^{-\mathrm{ad}\,\beta}\right) + \left(\delta\beta \sslash \dfrac{1 - e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}\right)$

21. ✓ The differential of C: $\quad \delta C_u^\gamma = \mathrm{ad}_u\left\{\delta\gamma \sslash \dfrac{e^{\mathrm{ad}\,\gamma} - 1}{\mathrm{ad}\,\gamma} \sslash RC_u^{-\gamma}\right\} \sslash C_u^\gamma$

22. ✓ The differential of $C_{u,v,\ldots}$: $\quad \delta C_{u,v,\ldots}^{\alpha,\beta,\ldots} = \mathrm{ad}_{u,v,\ldots}\left\{\delta\alpha \sslash \dfrac{e^{\mathrm{ad}\,\alpha} - 1}{\mathrm{ad}\,\alpha} \sslash RC_{u,v,\ldots}^{-\alpha,-\beta,\ldots}, \delta\beta \sslash \dfrac{e^{\mathrm{ad}\,\beta} - 1}{\mathrm{ad}\,\beta} \sslash RC_{u,v,\ldots}^{-\alpha,-\beta,\ldots}, \ldots\right\} \sslash C_{u,v,\ldots}^{\alpha,\beta,\ldots}$

23. ✓ The differential of RC: $\quad \delta RC_u^\gamma = RC_u^\gamma \sslash \mathrm{ad}_u\left\{\delta\gamma \sslash \dfrac{1 - e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} \sslash RC_u^\gamma\right\}$

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24. ✓ The differential of J: $\quad \delta J_u(\gamma) = \delta\gamma \sslash \dfrac{1 - e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} \sslash RC_u^\gamma \sslash \mathrm{div}_u \sslash C_u^{-\gamma}$