Stefan Sakalos: On Quantization of Quasi-Lie Bialgebras

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BiAlgebra: (B, Δ, ϵ) Example. $\mathcal{U}(\mathfrak{g})$

Monoidal categories:

Functor $\otimes : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$, unit 1, associativity.

Pentagon.

B-Mod is a monodal category.

Quasi-Bi-Algebra: $(B, \Delta, \epsilon, \underset{\text{invertible}}{\Phi \in B^{\otimes 3}})$

What should Φ satisfy to get a monoidal category?

Ans. The usual suspects.

A deformation of an algebra A: a product on $B := A[[\hbar]]$ s.t. $B/\hbar B \cong A$.

$$\{f, g\} := \frac{1}{\hbar}[f, g]_B \mod B$$

is a Poisson structure.

Given a Poisson structure, does it always have a quantization?

A deformation of the BiAlgebra $\mathcal{U}(\mathfrak{g})$ is a "QUE".

A deformation of $\mathcal{U}(\mathfrak{g})$ as a quasi-Hopf bialgebra is a "quasi QUE algebra". Get

$$\delta(x) := \frac{1}{\hbar}\Delta(x) \mod \hbar B$$

.... get Lie bialgebras.

A quantization of a Lie-bialgebra ....

exists by Etingof-Kazhdan.

Classical limit of a quasi-QUE:

get also $\varphi = \frac{1}{\hbar^2} \mathrm{Alt}\, \Phi \mod \hbar$

φ satisfies some relations....

set $\mathfrak{p} = \mathfrak{g} \oplus \mathfrak{g}^*$, has metric $\langle\ \rangle$.

A quasi Lie bialgebra structure on $\mathfrak{g}$ is the same as a Lie bracket on $\mathfrak{p}$ s.t.

1. $\mathfrak{g}$ is a Lie subalgebra.
2. $\langle\ \rangle$ is an invariant scalar product.

"The Drinfel'd double"

Theorem (Enriquez - Halbout). Quantizations here exist.

A Drinfel'd associator for a Lie algebra $\mathfrak{p}$ w/ metric t ....

Another method for quantization of quasi-Lie-bialgebras ... we construct quasi-bi-algebras as endomorphisms of "forgetful functors".
