Severa thoughts on December 12 - the (co)commutative case

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Severa's construction.
Given a Braided Monoidal Category (BMC) $\mathcal{D}$ (with Manin $(\partial, \mathfrak{g}, \mathfrak{g}^\star)$, set $\mathcal{D} := \mathcal{U}(\partial) - \mathrm{Mod}^{\Phi}$), given a co-braided co-algebra $(M, \Delta\colon M \to M^2, \epsilon\colon M \to 1_{\mathcal{D}})$ ($M := \mathcal{U}(\mathfrak{g}) = \mathcal{U}(\partial)/\mathcal{U}(\partial)\mathfrak{g}^*$), given a second BMC C (Vect), a functor $F\colon \mathcal{D} \to C$ ($F(X) := X/\mathfrak{g}X$) and a comonoidal structure c (namely a natural $c_{X,Y}\colon F(XY) \to F(X)F(Y)$ and $c_1\colon F(1_{\mathcal{D}}) \to 1_C$ respecting the braiding and associativity) such that

$$F(XMY) \xrightarrow{F(1\Delta 1)} F(XMMY) \xrightarrow{c_{XM,MY}} F(XM)F(MY)$$

$$\text{and} \quad F(M) \xrightarrow{F(\epsilon)} F(1_{\mathcal{D}}) \xrightarrow{c_1} 1_C$$

are isomorphisms (the clear $c_{X,Y}\colon XY/\mathfrak{g}(XY) \to (X/\mathfrak{g}X)(Y/\mathfrak{g}Y)$), construct a Hopf algebra structure on $H := F(M^2)$:

$$\Delta_H\colon\ F(M^2) \xrightarrow{F(\Delta\Delta)} F(M^4) \xrightarrow{F(1R1)} F(M^4) \xrightarrow{c_{M,M}} F(M^2)^2,$$

$$m_H\colon\quad F(M^2)^2 \xleftarrow[\sim]{c_{M^2,M^2}\circ F(1\Delta 1)} F(M^3) \xrightarrow{F(1\epsilon 1)} F(M^2),$$

$$S_H\colon\quad F(M^2) \xrightarrow{F(R)} F(M^2).$$

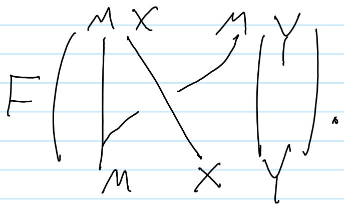
Set also $G\colon X \mapsto F(MX)$ ($G\colon X \mapsto \frac{\mathcal{U}(\mathfrak{g})X}{\mathfrak{g}(\mathcal{U}(\mathfrak{g})X)}$).

Questions. • Is H the symmetry algebra of something?
• In the non-quasi case, can we reconstruct $\mathcal{U}(\mathfrak{g})$ from the category of ∂-modules?
• In the abstract context, what is the relation between H and M?
• How does this restrict to AT/AET in the co-commutative case?

Now $M = \mathcal{U}(\mathfrak{g}) = \mathcal{U}(\partial)/\mathcal{U}(\partial)\mathfrak{g}$ $\qquad F(X) := X/\mathfrak{g}^* X$

$G(X) := F(MX)$

$G(XY) \xrightarrow{J} G(X)G(Y)$ by



To get J, take $X = Y = \mathcal{U}(\partial)$.

1. How is it an element of $\mathcal{U}(\partial)^{\otimes 2}$?
2. How is it an element of $TAut_2$?