Severa on quantization of Lie bialgebras

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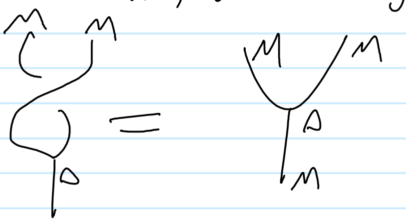
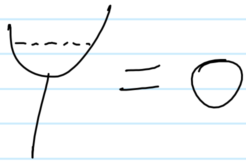
generalities	Example
$\mathcal{D}$ - BMC (Braided monoidal category)	$\mathfrak{g}, \mathfrak{g}^* \subset \partial$ $\mathcal{D} = U(\partial) - Mod^{\Phi}$ More precisely, $\mathfrak{g}$-modules that are also $\mathfrak{g}$-cocomdules.
$M \in \mathcal{D}$ $\quad \epsilon: M \to 1_{\mathcal{D}}$ $\Delta: M \to M \otimes M$ co-commutative coalgebra 	$M = U(\mathfrak{g}) = U(\partial)/U(\partial)\mathfrak{g}^*$ usual Δ, ϵ, satisfies  (diagrams are read from bottom to top)
$\mathcal{E}$ - BMC	$\mathcal{E} = Vect$
$F: \mathcal{D} \to \mathcal{E}$ $F(X \otimes Y) \longrightarrow F(X) \otimes F(Y)$ braided co-monoidal s.t. $F((X \otimes M) \otimes Y) \xrightarrow{\Delta} F(X \otimes M) \otimes F(M \otimes Y)$ is an isomorphism and $F(M) \xrightarrow{F(\epsilon)} F(1_{\mathcal{D}}) \to 1_{\mathcal{E}}$ (iso)	$F(X) = X/\mathfrak{g}X$ $X \otimes Y/\mathfrak{g}(X \otimes Y) \longrightarrow X/\mathfrak{g}X \otimes Y/\mathfrak{g}Y$
	$\mathcal{D} \xrightarrow[\text{comonoidal}]{M \otimes} \mathcal{D} \xrightarrow[\text{comonoidal}]{F} \mathcal{E}$ "The Twist" by $(M \otimes X) \otimes (M \otimes Y) \to M \otimes (X \otimes Y)$ composition should be strongly comonoidal.

In these circumstances,

$H := F(M \otimes M)$ $\qquad$ (in example, $\cong U(\mathfrak{g})$)

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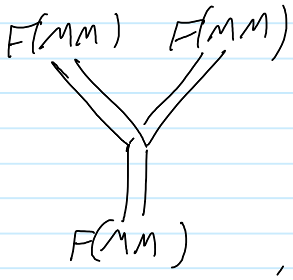
(in example, $\cong U(\mathfrak{g})$ by $x \otimes y \mapsto S(x)y$)

H is a co-algebra:



H is an algebra:

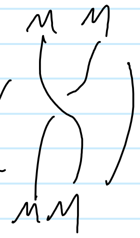
$$F(M\otimes M\otimes M\otimes M) \xrightarrow{F(1\otimes\varepsilon\otimes 1)} F(M\otimes M)$$

$$S\downarrow \qquad\qquad \nearrow \text{ product constructed.}$$

$$F(M\otimes M)\otimes F(M\otimes M)$$

Unit: $1_{\mathcal{E}} \cong F(M) \longrightarrow F(M\otimes M)$

Antipode:

$F($  $) = S$

Associativity by looking at 

$F(M\otimes M\otimes M\otimes M)$

$M^{\otimes(n+1)} =: X_n$ simplicial coalgebra in $\mathcal{D}$

Applying F we get a simplicial coalgebra in $\mathcal{E}$

Another example where construction applies:

∂, $\langle\ \rangle$

$\mathfrak{p}, \mathfrak{p}'$ co-isotropic

Example:

$\underbrace{n \otimes \overbrace{h}\otimes n'}$

$\mathfrak{p}$, $\mathfrak{p}'$

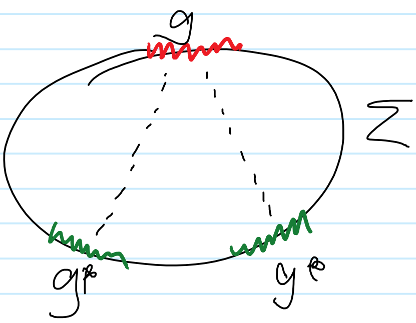
$\mathcal{D}$: $U(\mathfrak{g})$-Mod^{Φ}

$M = U(\mathfrak{n}) = U\partial / (U\partial)\mathfrak{p}'$

$F(X) = X/nX$

$F : \mathcal{D} \longrightarrow U(h)\text{-Mod}^{\otimes} =: \mathcal{C}$

Where it comes from?



$\longrightarrow$ $\mathcal{M} =$ Flat $\mathfrak{d}$-connections on Σ / constrained gauge

$(\mathfrak{g}, \mathfrak{g}^*) \subset \mathfrak{d}$ coisotropic

$\mathcal{M}$ is a Poisson manifold
in the case on the left it is G as a Poisson-Lie group.