Cheat Sheet Severa Quantization on Dec 4, 2013

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Severa's construction.

Given a Braided Monoidal Category (BMC) $\mathcal{D}$ (with Manin $(\partial, \mathfrak{g}, \mathfrak{g}^\star)$, set $\mathcal{D} := \mathcal{U}(\partial) - \mathrm{Mod}^\Phi$), given a co-braided co-algebra $(M, \Delta\colon M \to M \otimes M, \epsilon\colon M \to 1_\mathcal{D})$ ($M := \mathcal{U}(\mathfrak{g}) = \mathcal{U}(\partial)/\mathcal{U}(\partial)\mathfrak{g}^*$), given a second BMC C (Vect), a functor $F\colon \mathcal{D} \to C$ ($F(X) := X/\mathfrak{g}X$) and a comonoidal structure c (namely a natural $c_{X,Y}\colon F(XY) \to F(X)F(Y)$ respecting the braiding and associativity) such that

$$F(XMY) \xrightarrow{F(1\Delta 1)} F(XMMY) \xrightarrow{c} F(XM)F(MY)$$

and

$$F(M) \xrightarrow{F(\epsilon)} F(1_\mathcal{D}) \longrightarrow 1_C$$

are isomorphisms (the obvious $c_{X,Y}\colon XY/\mathfrak{g}(XY) \to (X/\mathfrak{g}X)(Y/\mathfrak{g}Y)$), construct a Hopf algebra structure on $H := F(M^2)$:

$$\Delta_H\colon\ F(M^2) \xrightarrow{F(\Delta\Delta)} F(M^4) \xrightarrow{F(1R1)} F(M^4) \xrightarrow{c_{M,M}} F(M^2)^2,$$

$$m_H\colon\ F(M^2)^2 \xleftarrow[\sim]{c_{M,M}\circ F(1\Delta 1)} F(M^3) \xrightarrow{F(1\epsilon 1)} F(M^2),$$

$$S_H\colon\ F(M^2) \xrightarrow{F(R)} F(M^2).$$

Set also $G\colon X \mapsto F(MX)$ ($G\colon X \mapsto \frac{\mathcal{U}(\mathfrak{g})X}{\mathfrak{g}(\mathcal{U}(\mathfrak{g})X)}$).

Questions. • Is H the symmetry algebra of something?
• In the non-quasi case, can we reconstruct $\mathcal{U}(\mathfrak{g})$ from the category of ∂-modules?
• In the abstract context, what is the relation between H and M?

Tannakian reconstruction.

— Given an algebra A let $\mathcal{D} := A - \mathrm{Mod}$ (projective (?) left A-modules), let $C :=$ Vect and $G\colon \mathcal{D} \to C$ be the forGetful functor. Then $A \simeq \mathrm{End}(G)$ by

$$a \in A \mapsto (\text{the action of } a \text{ on any } X \in \mathcal{D}),$$
$$\{h_X\colon G(X) \to G(X)\}_{X\in\mathcal{D}} \mapsto h_A(1) \in A.$$

— Given a monoidal $\mathcal{D}$ and an exact $G\colon \mathcal{D} \to C =:$ Vect with a natural isomorphism $\alpha_{X,Y}\colon G(X)G(Y) \to G(XY)$, there is a Hopf algebra structure on $H := \mathrm{End}(G)$: product is composition, coproduct $\Delta\colon H \to H \otimes H = \mathrm{End}(G^2\colon \mathcal{D}\times\mathcal{D} \to C)$ by

$$(h_X)_{X\in\mathcal{D}} \mapsto \left((X, Y) \mapsto \alpha_{X,Y} /\!/ h_{XY} /\!/ \alpha^{-1}_{X,Y} \in \mathrm{End}(G(X)G(Y))\right).$$