# Cheat Sheet Severa Quantization on Dec 3, 2013

December-03-13 11:41 AM

## Cheat Sheet Severa Quantization

http://drorbn.net/AcademicPensieve/2013-12/
initiated Dec 3, 2013; modified 3/12/13

**Tannakian reconstruction.**
— Given an algebra $A$ let $\mathcal{D} := A-\mathrm{Mod}$ (left $A$-modules), let $\mathcal{C} := \mathrm{Vect}$ and $F: \mathcal{D} \to \mathcal{C}$ be the forgetful functor. Then $A \simeq \mathrm{End}(F)$. $A$
— Given a monoidal $\mathcal{D}$ and an exact $F\colon \mathcal{D} \to \mathcal{C} =: \mathrm{Vect}$ with a natural isomorphism $\alpha_{X,Y}\colon F(X)F(Y) \to F(XY)$, there is a Hopf algebra structure on $H := \mathrm{End}(F)$: product is composition, coproduct $\Delta\colon H \to H \otimes H = \mathrm{End}(F^2\colon \mathcal{D}\times\mathcal{D} \to \mathcal{C})$ by

$$(h_X)_{X\in\mathcal{D}} \mapsto \big((X,Y) \mapsto \alpha_{X,Y}/\!/h_{XY}/\!/\alpha^{-1}_{X,Y} \in \mathrm{End}(F(X)F(Y))\big).$$

projective?

**Severa's construction.**
Given a Braided Monoidal Category (BMC) $\mathcal{D}$ (with Manin $(\partial, \mathfrak{g}, \mathfrak{g}^\star)$, set $\mathcal{D} := \mathcal{U}(\partial) - \mathrm{Mod}^\Phi$), given a co-braided co-algebra $(M, \Delta\colon M \to M\otimes M, \epsilon\colon M \to 1_{\mathcal{D}})$ ($M := \mathcal{U}(\mathfrak{g}) = \mathcal{U}(\partial)/\mathcal{U}(\partial)\mathfrak{g}^\star$), given a second BMC $\mathcal{C}$ (Vect), a functor $F\colon \mathcal{D} \to \mathcal{C}$ ($F(X) := X/\mathfrak{g}X$) and a comonoidal structure $c$ (namely a natural $c_{X,Y}\colon F(XY) \to F(X)F(Y)$ respecting the braiding and associativity) such that

$$F(XMY) \xrightarrow{1F(\Delta)1} F(XMMY) \xrightarrow{c} F(XM)F(MY)$$
$$F(M) \xrightarrow{F(\epsilon)} F(1_{\mathcal{D}}) \longrightarrow 1_{\mathcal{C}}$$

are isomorphisms (the obvious $c_{X,Y}\colon XY/\mathfrak{g}(XY) \to (X/\mathfrak{g}X)(Y/\mathfrak{g}Y)$), construct a Hopf algebra structure on $H := F(MM)$.

$H := \mathrm{End}(F) = F$

A: $A \cong \mathrm{End}\,F$: $\{h_X : F(X) \circlearrowleft\} \longmapsto h_A(1) \in A$

$a \in A \longmapsto$ the action of $a$, on any $X \in \mathcal{D}$

$$G(X) = F(MX) = \frac{\mathcal{U}(\partial)}{\mathcal{U}(\partial)\mathfrak{g}^*} \otimes X \Big/ \mathfrak{g}(-)$$

To do: In the non-quasi case, reconstruct $\mathcal{U}(\mathfrak{g})$ from the category of $\partial$-modules.

H is a co-algebra:

$$\begin{array}{ccc} F(MM) & & F(MM) \\ & \diagdown\ \diagup & \\ & F(MM) & \end{array}$$

H is an algebra:

$$\begin{tikzcd} F(M\otimes M\otimes M) \arrow[r, "F(1\otimes\epsilon\otimes 1)"] \arrow[d, "S"] & F(M\otimes M) \\ F(M\otimes M)\otimes F(M\otimes M) \arrow[ur, dashed] & \end{tikzcd}$$

product constructed.