

Cheat Sheet Severa Quantization

<http://drorbn.net/AcademicPensieve/2013-12/>
 initiated Dec 3, 2013; modified 3/12/13

Tannakian reconstruction.

— Given an algebra A let $\mathcal{D} := A - \text{Mod}$ (left A -modules), let $\mathcal{C} := \text{Vect}$ and $F : \mathcal{D} \rightarrow \mathcal{C}$ be the forgetful functor. Then $A \simeq \text{End}(F)$. A

— Given a monoidal \mathcal{D} and an exact $F : \mathcal{D} \rightarrow \mathcal{C} := \text{Vect}$

with a natural isomorphism $\alpha_{X,Y} : F(X)F(Y) \rightarrow F(XY)$, there is a Hopf algebra structure on $H := \text{End}(F)$: product is composition, coproduct $\Delta : H \rightarrow H \otimes H = \text{End}(F^2 : \mathcal{D} \times \mathcal{D} \rightarrow \mathcal{C})$ by $(h_X)_{X \in \mathcal{D}} \mapsto ((X, Y) \mapsto \alpha_{X,Y} // h_{XY} // \alpha_{X,Y}^{-1} \in \text{End}(F(X)F(Y)))$.

Severa's construction.

Given a Braided Monoidal Category (BMC) \mathcal{D} (with Manin (∂, g, g^*) , set $\mathcal{D} := \mathcal{U}(\partial) - \text{Mod}^\Phi$), given a co-braided co-algebra $(M, \Delta : M \rightarrow M \otimes M, \epsilon : M \rightarrow 1_{\mathcal{D}})$ ($M := \mathcal{U}(\mathfrak{g}) = \mathcal{U}(\partial)/\mathcal{U}(\partial)g^*$), given a second BMC \mathcal{C} (Vect), a functor $F : \mathcal{D} \rightarrow \mathcal{C}$ ($F(X) := X/gX$) and a comonoidal structure c (namely a natural $c_{X,Y} : F(XY) \rightarrow F(X)F(Y)$ respecting

the braiding and associativity) such that

$$F(XMY) \xrightarrow{1_{F(\Delta)}} F(XMMY) \xrightarrow{c} F(XM)F(MY)$$

$$F(M) \xrightarrow{F(\epsilon)} F(1_{\mathcal{D}}) \xrightarrow{1_{\mathcal{C}}} 1_{\mathcal{C}}$$

are isomorphisms (the obvious $c_{X,Y} : XY/g(XY) \rightarrow (X/gX)(Y/gY)$), construct a Hopf algebra structure on $H := F(MM)$.

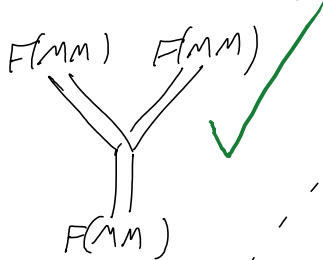
$H := \text{End}(F) = F$

$A \simeq \text{End } F : \{h_X : F(X) \rightarrow F(X)\} \mapsto h_A(1) \in A \checkmark$
 $a \in A \mapsto$ the action of a , on any $X \in \mathcal{D} \checkmark$

$G(X) = F(MX) = \frac{\mathcal{U}(\mathfrak{g})}{\mathcal{U}(\mathfrak{g})g^*} \otimes X / g(-) \checkmark$

To do: In the non-quasi case, reconstruct $\mathcal{U}(\mathfrak{g})$ from the category of ∂ -modules.

H is a co-algebra:



H is an algebra:

