Cheat Sheet J

http://drorbn.net/AcademicPensieve/2013-12/
initiated 18/3/13; continues 2013-03; continued 2014-01; modified 8/1/14, 2:28pm

With alphabet T and with $u,v,w\in T$, $\alpha,\beta,\gamma\in FL(T)$, $D\in\mathfrak{tder}(T)$, $g,h\in\exp(\mathfrak{tder}(T))=\mathrm{TAut}(T)$. Checkmarks (✓) as in `CheatSheetJ-Verification.nb`.

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1. The definition of J: $$J_u(\gamma) := \int_0^1 ds\ \mathrm{div}_u(\gamma /\!/ RC_u^{s\gamma}) /\!/ C_u^{-s\gamma}$$
2. ✓ The J_{uv} equation: $$J_u(\alpha)+J_v(\beta/\!/RC_u^\alpha)/\!/C_u^{-\alpha} = J_v(\beta)+J_u(\alpha/\!/RC_v^\beta)/\!/C_v^{-\beta}$$
3. ✓ The t equation: $$J_w(\gamma/\!/tm_w^{uv}) = \left(J_u(\gamma)+J_v(\gamma/\!/RC_u^\gamma)/\!/C_u^{-\gamma}\right)/\!/tm_w^{uv}$$
4. ✓ The h equation: $$J_u(\mathrm{bch}(\alpha,\beta)) = J_u(\alpha)+J_u(\beta/\!/RC_u^\alpha)/\!/C_u^{-\alpha}$$

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5. ✓ The meaning(s) of RC: $C_u^\gamma /\!/ RC_u^{-\gamma} = Id$, $\qquad C_u^{\gamma/\!/RC_u^\gamma} = RC_u^\gamma$
6. ✓ C_uC_v and RC_uRC_v: $C_u^{\alpha/\!/RC_v^{-\beta}} /\!/ C_v^\beta = C_v^{\beta/\!/RC_u^{-\alpha}} /\!/ C_u^\alpha$, $\qquad RC_u^\alpha /\!/ RC_v^{\beta/\!/RC_u^\alpha} = RC_v^\beta /\!/ RC_u^{\alpha/\!/RC_v^\beta}$
7. RC equation t: $$tm_w^{uv} /\!/ RC_w^{\gamma/\!/tm_w^{uv}} = RC_u^\gamma /\!/ RC_v^{\gamma/\!/RC_u^\gamma} /\!/ tm_w^{uv}$$
8. RC equation h: $$RC_u^{\mathrm{bch}(\alpha,\beta)} = RC_u^\alpha /\!/ RC_u^{\beta/\!/RC_u^\alpha}$$
9. C–div–RC equations: $\mathrm{div}_u(\alpha/\!/RC_u^\gamma)/\!/C_u^\gamma = ?$ $\qquad \mathrm{div}_u(\alpha/\!/C_u^\gamma)/\!/RC_u^\gamma = ?$

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10. div property t: $$\mathrm{div}_w(\gamma/\!/tm_w^{uv}) = (\mathrm{div}_u(\gamma)+\mathrm{div}_v(\gamma))/\!/tm_w^{uv}$$
11. ✓ div property uv: with $\mathrm{ad}_u^\gamma = \mathrm{ad}_u\{\gamma\} := \mathrm{der}(u\to[\gamma,u])$, $$(\mathrm{div}_u\,\alpha)/\!/\mathrm{ad}_v^\beta - (\mathrm{div}_v\,\beta)/\!/\mathrm{ad}_u^\alpha = \mathrm{div}_u(\alpha/\!/\mathrm{ad}_v^\beta) - \mathrm{div}_v(\beta/\!/\mathrm{ad}_u^\alpha)$$
12. ✓ div property uu: $$(\mathrm{div}_u\,\alpha)/\!/\mathrm{ad}_u\{\beta\} - (\mathrm{div}_u\,\beta)/\!/\mathrm{ad}_u\{\alpha\} = \mathrm{div}_u\left([\alpha,\beta]+\alpha/\!/\mathrm{ad}_u\{\beta\}-\beta/\!/\mathrm{ad}_u\{\alpha\}\right)$$

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13. The definition of JA: $$JA_u(\gamma) := J_u(\gamma)/\!/RC_u^\gamma$$
14. The ODE for JA: with $\gamma_s = \gamma/\!/RC_u^{s\gamma}$, $$JA(0)=0,\quad \frac{dJA(s)}{ds} = JA(s)/\!/\mathrm{ad}_u\{\gamma_s\}+\mathrm{div}_u\,\gamma_s,\quad JA(1)=JA_u(\gamma)$$

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15. The relation with $\mathfrak{tder}$, 1: with $\gamma' = \gamma/\!/e^{\mathrm{ad}_u\{\gamma\}}$, $\qquad e^{\mathrm{ad}_u\{\gamma\}} = C_u^{\gamma'}$???
16. The relation with $\mathfrak{tder}$, 2: $\qquad C_u^\gamma = e^{\mathrm{ad}_u\{?\}}$

(there should be a many-variable version of the above two cheatlines)

17. The definition of j (following A-T): $$j(e^D) = \int_0^1 ds\, e^{sD}(\mathrm{div}\,D) = \frac{e^D-1}{D}(\mathrm{div}\,D)$$
18. j's cocycle property: $$j(gh) = j(g)+g\cdot j(h)$$

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19. The differential of exp: $$\delta e^\gamma = e^\gamma\cdot\left(\delta\gamma/\!/\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right) = \left(\delta\gamma/\!/\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right)\cdot e^\gamma$$
20. ✓ The differential of $\gamma = \mathrm{bch}(\alpha,\beta)$: $$\delta\gamma/\!/\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} = \left(\delta\alpha/\!/\frac{1-e^{-\mathrm{ad}\,\alpha}}{\mathrm{ad}\,\alpha}/\!/e^{-\mathrm{ad}\,\beta}\right)+\left(\delta\beta/\!/\frac{1-e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}\right)$$
21. ✓ The differential of C: $$\delta C_u^\gamma = \mathrm{ad}_u\left\{\delta\gamma/\!/\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}/\!/RC_u^{-\gamma}\right\}/\!/C_u^\gamma$$
22. ✓ The differential of RC: $$\delta RC_u^\gamma = RC_u^\gamma/\!/\mathrm{ad}_u\left\{\delta\gamma/\!/\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}/\!/RC_u^\gamma\right\}$$

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23. ✓ The differential of J: $$\delta J_u(\gamma) = \delta\gamma/\!/\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}/\!/RC_u^\gamma/\!/\mathrm{div}_u/\!/C_u^{-\gamma}$$